

**THE PHYSICS OF FLUIDS IN HIERARCHICAL POROUS MEDIA:  
ANGSTROMS TO MILES**

# Theory and Applications of Transport in Porous Media

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# The Physics of Fluids in Hierarchical Porous Media: Angstroms to Miles

by

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*This text is dedicated to Ruth  
and to the memory of Rachel*

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## Preface

Porous media are ubiquitous throughout nature and many modern technologies. Examples of natural porous media include tissues, cells, folded proteins, whole plants and animals, soils, aquifers, and reservoirs. Modern technologies involve pores and porous media in many different ways. For example, they are involved with gel electrophoresis, chromatography, the atomic force and scanning tunneling microscopes, the formation of composites, drug delivery substrates, protective clothing, insulation (ceramics and fiberglass), air filters, ion exchange columns, and etc. When viewed on an appropriate scale, almost anything can be thought of as being porous. For this reason, the scale of observation is critically important in theories of flow and deformation in porous media.

Because of their omnipresent nature, porous media are studied to one degree or another in almost all branches of science and engineering. And not surprisingly, this text is an outgrowth of a two-semester advanced graduate level course offered to applied mathematicians, physicists, chemists, engineers (chemical, civil, mechanical, and agricultural) and environmental and soil scientists at Purdue University. Its contents result from my attempt to develop a coherent and rational multiscale approach for studying porous media and fluids therein. While many of the problems studied are restrictive, the tools and techniques developed and used are generic, and as such are applicable to a much wider range of problems and topics than those presented. No attempt is made to survey the broad literature on porous media, rather the problems studied are based on the efforts of my group over approximately the last five years. The reader is assumed to have familiarity with mathematics through a first course in PDE's and an introduction to stochastic processes. Most requisite background material is contained within the text.

Chapters I and II provide many of the tools which are prerequisites for later chapters. Chapter I presents an elementary background in continuum physics for single phases (classical continuum mechanics), species within a phase (mixture theory), and mixtures of phases and species (hybrid mixture theory). Much of the early material in this chapter is condensed from Eringen [65]. Chapter II summarizes both classical equilibrium and nonequilibrium statistical mechanics. In addition, it introduces the reader to nonlocality and to the duality between continuum and statistical mechanical constructs.

Chapters III and IV focus on the anomalous behavior of fluids in microporous materials. A micropore can be defined as a pore with at least one characteristic dimension on the order of a few fluid molecular diameters. Consequently, the size of a micropore is very "small" if the fluid molecule

is small, but it can be rather “large” if the molecule is large. We focus on the former of these scenarios. Such micropores are commonly found in swelling colloids such as bentonite clays and polymers. Technologies involving micropores include atomic force microscopy (AFM) and scanning tunneling microscopy (STM). Fluid behavior in microporous media is incredibly rich and different than its bulk phase counterpart with which it may be in equilibrium. Using statistical mechanical tools it is illustrated how substantially these fluids differ from their bulk counterparts.

Chapter V deals with the propagation of information from the “microscale” to the “mesoscale” and the mesoscale to the “macroscale” for systems with colloids. Here, in the case of swelling clays for example, the microscale is defined as the scale of the individual clay platelet or the fluid solvating the platelet (vicinal fluid). The mesoscale is a homogenization of the platelets and vicinal water to form a clay particle (mixture of platelets and vicinal water viewed as a particle). The macroscale is a homogenization of the particles with bulk water. Applications to the drying of shrinking biogels and the consolidation of clay soils are presented.

Chapter VI deals with natural porous media, on scales of meters to miles, where there may be no discrete spatial or temporal scale of motion. The main tool used here is perturbation theory applied to stochastic PDE's. The practical problem studied is the evolution of dissolved contaminants in natural geologic media. The perturbation results are compared to extensive Monte Carlo simulations over random fields.

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