

NUMERICAL METHODS FOR SHALLOW-WATER FLOW

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NUMERICAL METHODS FOR SHALLOW-WATER FLOW

by

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Preface

Unlike many other books, this one did not grow out of lecture notes. Rather, I felt that after more than 25 years of widespread application of numerical shallow-water models, a suitable reference book was still missing. Of course, sections on shallow-water flow are included in some CFD books, but until very recently a systematic treatment of both physical and numerical aspects was not available. Most of the literature is scattered over hundreds of papers and reports.

In this text, the theory of shallow-water flow and its numerical simulation is given. I am stressing the physics in the first part of the book because I am convinced that you need to realize which type of solutions to expect. In the second part, typical numerical methods are discussed. This is not a cook-book and I did not include all methods available in literature, but I emphasize how to obtain guidelines for choosing a method for your particular problem, taking such things as stability and, most importantly, accuracy into account.

Most of the book is about two-dimensional flows, the treatment of which is well-established. Recent research, however, is concentrating on 3-d flows. Therefore one chapter has been included which discusses the main aspects of such flows. Case studies are not given. My experience is that published case studies rarely provide sufficient details to be really useful. Moreover, giving case studies for the whole spectrum of applications and the whole spectrum of numerical methods would take far too much space.

In selecting material, I have been slightly biased towards work from The Netherlands. There is some justification for this, as significant contributions have come from this country and from persons closely related with it. Nevertheless, I have attempted to include all relevant contributions, though without any claim to be exhaustive.

The audience of this book is supposed to consist of three (perhaps not disjoint) groups. First of all, I have in mind scientists and engineers involved in applications of shallow-water models; they should have a sufficient knowledge of physics and numerics to judge the reliability of their results. Secondly, developers of fluid-dynamics software should be aware of what has been accomplished and where the problems are. Finally, numerical mathematicians need insight in the type of problems they are developing solution methods for. Although the field of applications is very wide (see chapter 1) and I have tried to do justice to it, the major emphasis is on engineering applications in rivers, estuaries and coastal seas. The book is self-contained but the reader is assumed to have a basic knowledge of fluid mechanics and CFD.

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