Solving Frontier Problems of Physics:
The Decomposition Method

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Solving Frontier Problems of Physics: The Decomposition Method

by

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IN MEMORY OF MY FATHER AND MOTHER
HAIG AND VARTUHI ADOMIAN

EARLIER WORKS BY THE AUTHOR

Applied Stochastic Processes, Academic Press, 1980.
Stochastic Systems, Academic Press, 1983; also Russian transl. ed. H.G.Volkova, Mir Publications, Moscow, 1987.
Partial Differential Equations with R. E. Bellman, D. Reidel Publishing Co., 1985.
Nonlinear Stochastic Operator Equations, Academic Press, 1986.
Nonlinear Stochastic Systems Theory and Applications to Physics, Kluwer Academic Publishers, 1989.

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PREFACE

I discovered the very interesting Adomian method and met George Adomian himself some years ago at a conference held in the United States. This new technique was very surprising for me, an applied mathematician, because it allowed solution of exactly nonlinear functional equations of various kinds (algebraic, differential, partial differential, integral,...) without discretizing the equations or approximating the operators. The solution when it exists is found in a rapidly converging series form, and time and space are not discretized. At this time an important question arose: why does this technique, involving special kinds of polynomials (Adomian polynomials) converge? I worked on this subject with some young colleagues at my research institute and found that it was possible to connect the method to more well-known formulations where classical theorems (fixed point theorem, substituted series, ...) could be used. A general framework for decomposition methods has even been proposed by Lionel Gabet, one of my researchers who has obtained a Ph.D. thesis on this subject. During this period a fruitful cooperation has been developed between George Adomian and my research institute. We have frequently discussed advances and difficulties and we exchange ideas and results.

With regard to this new book, I am very impressed by the quality and the importance of the work, in which the author uses the decomposition method for solving frontier problems of physics. Many concrete problems involving differential and partial differential equations (including Navier-Stokes equations) are solved by means of the decomposition technique developed by Dr. Adomian. The basic ideas are clearly detailed with specific physical examples so that the method can be easily understood and used by researchers of various disciplines. One of the main objectives of this method is to provide a simple and unified technique for solving nonlinear functional equations.

Of course some problems remain open. For instance, practical convergence may be ensured even if the hypotheses of known methods are not satisfied. That means that there still exist opportunities for further theoretical studies to be done by pure or applied mathematicians, such as proving convergence in more general situations. Furthermore, it is not always easy to take into account the boundary conditions for complex domains.

In conclusion, I think that this book is a fundamental contribution to the theory and practice of decomposition methods in functional analysis. It completes and clarifies the previous book of the author published by Kluwer in 1989. The decomposition method has now lost its mystery but it has won in seriousness and power. Dr. Adomian is to be congratulated for his fundamental contribution to functional and numerical analysis of complex systems.

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September 9, 1993

FOREWORD

This book is intended for researchers and (primarily graduate) students of physics, applied mathematics, engineering, and other areas such as biomathematics and astrophysics where mathematical models of dynamical systems require quantitative solutions. A major part of the book deals with the necessary theory of the decomposition method and its generalizations since earlier works. A number of topics are not included here because they were dealt with previously. Some of these are delay equations, integro-differential equations, algebraic equations and large matrices, comparisons of decomposition with perturbation and hierarchy methods requiring closure approximation, stochastic differential equations, and stochastic processes [1]. Other topics had to be excluded due to time and space limitations as well as the objective of emphasizing utility in solving physical problems.

Recent works, especially by Professor Yves Cherruault in journal articles and by Lionel Gabet in a dissertation, have provided a rigorous theoretical foundation supporting the general effectiveness of the method of decomposition. The author believes that this method is relevant to the field of mathematics as well as physics because mathematics has been essentially a linear operator theory while we deal with a nonlinear world. Applications have shown that accurate and easily computed quantitative solutions can be determined for nonlinear dynamical systems without assumptions of "small" nonlinearity or computer-intensive methods.

The evolution of the research has suggested a theory to unify linear and nonlinear, ordinary or partial differential equations for solving initial or boundary-value problems efficiently. As such, it appears to be valuable in the background of applied mathematicians and theoretical or mathematical physicists. An important objective for physics is a methodology for solution of dynamical systems—which yields verifiable and precise quantitative solutions to physical problems modelled by nonlinear partial differential equations in space and time. Analytical methods which do not require a change of the model equation into mathematically more tractable, but necessarily less realistic representation, are of primary concern. Improvement of analytical methods would in turn allow more sophisticated modelling and possible further progress. The final justification of theories of physics is in the correspondence of predictions with nature rather than in rigorous proofs which may well restrict the stated problem to a more limited universe. The broad applicability of the methodology is a dividend which may allow a new approach to mathematics courses as well as being useful for the physicists who will shape our future understanding of the world.

Recent applications by a growing community of users have included areas such as biology and medicine, hydrology, and semiconductors. In the author's opinion this method offers a fertile field for pure mathematicians and especially for doctoral students looking for dissertation topics. Many possibilities are included directly or indirectly. Some repetition of objectives and motivations (for research on decomposition and connections with standard methods) was believed to be appropriate to make various chapters relatively independent and permit convenient design of courses for different specialties and levels.

Partial differential equations are now solved more efficiently, with less computation, than in the author's earlier works. The Duffing oscillator and other generic oscillators are dealt with in depth. The last chapter concentrates on a number of frontier problems. Among these are the Navier-Stokes equations, the N-body problem, and the Yukawa-coupled Klein-Gordon-Schrödinger equation. The solutions of these involve no linearization, perturbation, or limit on stochasticity. The Navier-Stokes solution [2] differs from earlier analyses [3]. The system is fully dynamic, considering pressure changing as the velocity changes. It now allows high velocity and possible prediction of the onset of turbulence.

The references listed are not intended to be an exhaustive or even a partial bibliography of the valuable work of many researchers in these general areas. Only those papers are listed which were considered relevant to the precise area and method treated. (New work is appearing now at an accelerating rate by many authors for submission to journals or for dissertations and books. A continuing bibliography could be valuable to future contributors and reprints received by the author will be recorded for this purpose.)

The author appreciates the advice, questions, comments, and collaboration of early workers in this field such as Professors R.E. Bellman, N. Bellomo, Dr. R. MCarty, and other researchers over the years, the important work by Professor Yves Cherruault on convergence and his much appreciated review of the entire manuscript, the support of my family, and the editing and valuable contributions of collaborator and friend, Randolph Rach, whose insights and willingness to share his time and knowledge on difficult problems have been an important resource. The book contains work originally typeset by Arlette

Revells and Karin Haag. The camera-ready manuscript was prepared with the dedicated effort of Karin Haag, assisted by William David. Laura and William David assumed responsibility for office management so that research results could be accelerated. Computer results on the Duffing equation were obtained by Dr. McLowery Elrod with the cooperation of the National Science Center Foundation headed by Dr. Fred C. Davison, who has long supported this work. Gratitude is due to Ronald E. Meyers, U.S. Army Research Laboratories, White Sands Missile Range, who supported much of this research and also contributed to some of the development. Thanks are also due to the Office of Naval Research, Naval Research Laboratories, and Paul Palo of the Naval Civil Engineering Laboratories, who have supported work directed toward applications as well as intensive courses at NRL and NCEL. The author would also like to thank Professor Alwyn Van der Merwe of the University of Denver for his encouragement that led to this book. Most of all, the unfailing support by my wife, Corinne, as well as her meticulous final editing, is deeply appreciated.

G. Adomian

REFERENCES

1. G. Adomian, Stochastic Processes, *Encyclopedia of Sciences and Technology*, **16**, 2nd ed., Academic Press (1992).
2. G. Adomian, An Analytic Solution to the Stochastic Navier-Stokes System, *Foundations of Physics*, **2**, (831-834) (July 1991).
3. G. Adomian, *Nonlinear Stochastic Systems Theory and Applications to Physics*, Kluwer (192-216) (1989).