

## VIBRATION ANALYSIS OF ROTORS

# SOLID MECHANICS AND ITS APPLICATIONS

## Volume 21

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# Vibration Analysis of Rotors

by

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To  
my parents,  
my wife, Namdeuk Woo,  
and  
my children, Boramee and Seongwoo.

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## PREFACE

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This text is intended for use as an advanced course in either rotordynamics or vibration at the graduate level. This text has mostly grown out of the research work in my laboratory and the lectures given to graduate students in the Mechanical Engineering Department, KAIST. The text contains a variety of topics not normally found in rotordynamics or vibration textbooks. The text emphasizes the analytical aspects and is thus quite different from conventional rotordynamics texts; potential readers are expected to have a firm background in elementary rotordynamics and vibration.

In most previously published rotordynamics texts, the behavior of simple rotors has been of a primary concern, while more realistic, multi-degree-of-freedom or continuous systems are seldom treated in a rigorous way, mostly due to the difficulty of a mathematical treatment of such complicated systems. When one wanted to gain a deep insight into dynamic phenomena of complicated rotor systems, one has, in the past, either had to rely on computational techniques, such as the transfer matrix and finite element methods, or cautiously to extend ideas learned from simple rotors whose analytical solutions are readily available. The former methods are limited in the interpretation of results, since the calculations relate only to the simulated case, not to more general system behavior. Ideas learned from simple rotors can, fortunately, often be extended to many practical rotor systems, but there is of course no guarantee of their validity. This text is intended to include all possible analytical solutions to discrete or continuous rotor systems; it was stimulated from acquaintance with the monumental work, "Flexural Vibrations of Rotating Shafts," by F. M. Dimentberg.

Although most of topics treated in rotordynamics are closely tied with 'vibration,' they are not treated as in conventional vibration textbooks. It is the intent of this text to study 'rotor dynamics' from the vibration point of view. In this way, those who have been educated with a strong background in vibration can see the bridge between 'rotordynamics' and 'vibration', and can easily extend the knowledge gained from one area in the other.

The contents of the text are as follows:

The first chapter deals with the whirling phenomena of simple rotors. These are essential for an understanding of the dynamics and stability of rotating machinery, although the simple rotor models do not exhibit the effects of rotary inertia and gyroscopic moments. This introductory chapter also introduces concepts such as forward/backward modes, forward/backward whirls, synchronous/asynchronous whirls, the transformation between the stationary and rotating

coordinates, and the definitions of anisotropic/ asymmetric rotors.

In chapter 2, the simple rotor models treated in chapter 1 are extended to account for the gyroscopic and rotary inertia effects in order to investigate gyroscopic whirling characteristics. In particular an explanation is given for the separation of the forward and backward modal frequencies as the rotational speed changes.

Chapter 3 develops the linearized stiffness and damping coefficients of fluid film bearings, and treats the instability problem of oil whirl/whip, associated with the rotors supported in fluid film bearings.

Chapter 4 summarizes the useful definitions and theorems associated with lambda matrices and the general eigenvalue problem; this forms the basis of the theories developed in chapters 5, 6 and 7.

Chapter 5 develops the modal analysis method for multi-degree-of-freedom rotor-bearing systems and the complex modal testing theory which is a powerful tool for modal parameter identification of rotating machinery. Some of the subjects already treated in early chapters are re-examined to help the readers become familiar with the new approach based on lambda matrices.

In chapter 6, the definitions and theorems introduced in chapter 4 are used to develop the generalized modal analysis method for rotor-bearing systems with rotational speed dependent parameters. The rotational speed dependent phenomena occur because of the internal/external dampings (treated in chapter 1), gyroscopic moments (treated in chapter 2), and fluid film type of bearings (treated in chapter 3). This chapter emphasizes the method which transforms the rotational speed dependent eigenvalue problem to the standard eigenvalue problem independent of the rotational speed so that the vibration response to synchronous/asynchronous harmonic excitation can be directly obtained. In addition, the well-known balancing theories such as balancing using influence coefficients and modal balancing are developed in a rigorous way, based on the generalized modal analysis and the concept of phasor which is treated in chapter 1.

Chapter 7 contains the complete theory for the modal analysis of distributed parameter rotor-bearing systems. In particular, the modal frequencies, mode shapes, critical speeds, and stability of a rotating shaft with uniformly distributed shaft mass and disks along the length of the shaft are discussed, taking the rotary inertia and gyroscopic effects and the boundary conditions into consideration.

Chapter 8 deals with the advanced vibration theory of rotors under combined effects. The problems considered are: the effect of shear deformation, uneven speed, and torque on the vibration characteristics, and the dynamics of a cracked shaft.

The key ideas adopted in this text are:

- 1) Extensive use of complex notation, wherever applicable, to simplify the formulations and to preserve the directivity information of modes.
- 2) Introduction of lambda-matrices, which allow a unified approach to general vibration problems.
- 3) Gradual transition in the treatment of topics, from simple systems to complex ones.

The concept of a lambda-matrix has traditionally been set aside from the main stream of rotordynamics and vibration areas, mainly because of the unfamiliarity of the subject to many people. However, it has been shown in chapters 5, 6 and 7 that lambda-matrices can indeed be a powerful tool in analyzing the vibration of rotor-

bearing systems as the system complexity increases. On the other hand, the fundamental behaviors of simple systems such as the ones treated in chapters 1, 2 and 3 can be effectively investigated with simple mathematics. The essential role of the early chapters is to uncover the physical understanding of the vibration phenomena. To meet these seemingly contradictory requirements, the concept of a lambda-matrix is introduced in chapter 4 and the subjects treated in early chapters are re-examined in chapter 5, in the light of the new concept. It is hoped that, in this way, 'vibration' as well as 'rotordynamics' can be enriched both in scope and in depth.

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