MODEL THEORY FOR MODAL LOGIC

SYNTHESE LIBRARY

STUDIES IN EPISTEMOLOGY,

LOGIC, METHODOLOGY, AND PHILOSOPHY OF SCIENCE

VOLUME 127

KENNETH A. BOWEN
Syracuse University

MODEL THEORY FOR MODAL LOGIC

Kripke Models for Modal Predicate Calculi

Springer-Science+Business Media, B.V.

Library of Congress Cataloging in Publication Data

Bowen, Kenneth A.
Model theory for modal logic.

(Synthese library; v. 127)
Bibliography: p.
Includes indexes.
1. Modality (Logic) 2. Model theory.
3. Predicate calculus. I. Title. II. Title:
Kripke models for modal predicate calculi.
QA9.46.B68 511'.3 78-13182

ISBN 978-90-481-8353-1 ISBN 978-94-015-7642-0 (eBook)
DOI 10.1007/978-94-015-7642-0

Originally published by D. Reidel Publishing Company, Dordrect, Holland in 1979.
Softcover reprint of the hardcover 1st edition 1979

To Johanna

TABLE OF CONTENTS

PREFACE

Modal considerations in logic appeared in the work of the ancients, notably Aristotle, and the mediaeval logicians, but like most work before the modern period, it was non-symbolic, and not particularly systematic in approach. According to Hughes and Cresswell (1968), the earliest symbolic approaches to modal logic were those of MacColl, originating about 1880. However, the first symbolic *and* systematic approach to the subject appears to be the work of Lewis beginning in 1912 and culminating in the book *Symbolic Logic* with Langford in 1932. Since then a plethora of modal systems have been proposed, though Lewis' systems S1–S5 and the system called T by Feys (1937) and M by von Wright (1951) stand out as having received considerable attention.

By far the early emphasis was on propositional systems of modal logic (with the notable exception of the work of R. Barcan Marcus (1946a), (1946b) and (1947)) and prior to the 1950's the formal semantic work done had been of a topological or algebraic cast. The work of Kripke beginning in (1959) provided a structural semantics for many of these systems which, when applied to systems containing predicates and quantifiers applied to individual variables, provides a semantics whose general appearance is similar to that which has been constructed for classical predicate logic. Related work was begun about the same time by Hintikka (1961) and Montague (1960). With the exception of some work of Gabbay (1972a) and Osswald (1969) most of this formal work (as opposed to related philosophical applications) has consisted in establishing a semantics for the system and giving proofs of completeness relative to this semantics (cf. Føllesdal (1965), (1968), van Fraasen (1969), Hintikka (1961), (1963), (1967), Kripke (1959), (1963a), (1963b), (1965), Lemmon (1966), Makinson (1966), Routley (1970), Schütte (1970), Segerberg (1971), and Thomason (1970)). In Bowen (1975) it was shown that a considerable portion of the model theory of classical predicate logic could be transferred to the domain of normal modal logics (cf. §1), usually without serious distortion. Our purpose here is not only to give a systematic presentation of these results and further extensions, but also to show that the restriction to normal modal systems can largely be removed. However, we will still adopt the converse of the

Barcan formula (cf. §1); in fact, in the natural formulation of the systems considered, it is a provable statement. In §§1–4 we formulate the systems and their semantics. Since much of the previous work is scattered or inaccessible (notably Lemmon and Scott (1966)) we give a detailed presentation of the proofs of correctness and completeness in §§3–4. Model theory proper begins in §5.

This book grew out of a series of lectures given at the Stephan Banach International Mathematical Center in Warsaw during the spring of 1973, during which I was on sabbatical leave from Syracuse University and was also supported by the Center. I would especially like to thank all the participants in Prof. Rasiowa's seminar for their patience in my lectures and their many corrections and suggestions. Ester Clark persevered through the typing of the original manuscript, and Ruth Turnpaugh typed the appendix. For their patience I am most grateful. Much of the work presented here was partially supported by ARPA grant number DAHCO4-72-C-0003.