

## THE LOGIC OF CONDITIONALS

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ERNEST W. ADAMS

# THE LOGIC OF CONDITIONALS

*An Application of Probability to Deductive Logic*



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*To Anne, Jimmy, and Billy*

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## PREFACE

Of the four chapters in this book, the first two discuss (albeit in considerably modified form) matters previously discussed in my papers 'On the Logic of Conditionals' [1] and 'Probability and the Logic of Conditionals' [2], while the last two present essentially new material. Chapter I is relatively informal and roughly parallels the first of the above papers in discussing the basic ideas of a probabilistic approach to the logic of the indicative conditional, according to which these constructions do not have truth values, but they do have probabilities (equal to conditional probabilities), and the appropriate criterion of soundness for inferences involving them is that it should not be possible for all premises of the inference to be probable while the conclusion is improbable. Applying this criterion is shown to have radically different consequences from the orthodox 'material conditional' theory, not only in application to the standard 'fallacies' of the material conditional, but to many forms (e.g., Contraposition) which have hitherto been regarded as above suspicion. Many more applications are considered in Chapter I, as well as certain related theoretical matters. The chief of these, which is the most important new topic treated in Chapter I (i.e., this topic has not been treated in my own earlier articles), is a discussion of the fundamentally important *triviality results* of David Lewis ([40], as yet, alas, unpublished, in spite of the fact that these results must be central to any probabilistic approach to logic). What these results imply is that if the assumptions of the probabilistic theory are right, then *no* purely truth-conditional 'logic' of the conditional can avoid difficulties of the sort arising in the fallacies of material implication, and an adequate theory of the conditional *must* consider other 'dimensions of rightness' besides truth, and other criteria of soundness besides the classical one that the truth of premises should be inconsistent with the falsity of conclusions.

Chapter II, which is the only chapter of the book involving original mathematics, parallels my earlier paper 'Probability and the Logic of Conditionals' in proving a number of general theorems concerning the

properties of the probabilistic soundness criterion – that it should be impossible for the premises of an inference to be probable while its conclusion is improbable, the failure to satisfy which is what is wrong in the fallacies of material implication. The only thing to note about the present formulation is that the proofs have been radically simplified, essentially following the lines of related arguments given in my paper ‘The Logic of “Almost All”’ [4].

Chapter III is an attempt to argue for the rightness of the basic assumptions of the probabilistic theory (which entail the triviality results in turn), and to argue for the mistakenness of the assumptions of orthodox logic as it applies to conditionals. This argument involves what I regard as the most important new ideas in the present book, though these are probably the ones which will be least sympathetically received either by orthodox logicians or by the new breed of ‘philosophical logicians’. What I try to show is that probabilistic theory meets but orthodox theory fails to meet a *pragmatic requirement of adequacy* for theories of truth and soundness: namely, that it should be possible to demonstrate that persons are best advised to try to arrive at conclusions which are ‘true’ according to the tenets of the theory, and are best off reasoning in accord with principles which the theory holds to be sound. Without going into detail, an example from Section III.5 illustrates the failure of orthodox logic’s assumed *material truth* definition (giving the truth conditions for conditionals) to meet this requirement. Imagine a man about to eat some very good and non-poisonous mushrooms who is informed “if you eat those mushrooms you will be poisoned”, which leads the man not to eat the mushrooms, while making the statement ‘true’ (i.e., materially true) at the same time. Obviously the man would have been better off *not* to have arrived at this allegedly ‘true’ conclusion, and this type of example should make it questionable that reasoners should *want* to be guided in their reasoning by the principles of orthodox logic, if those are designed to lead them to conclusions which are ‘true’ in this unwanted sense. The positive argument of Chapter III is to show (at least in limited circumstances) that the proposed probabilistic theory does satisfy the pragmatic requirement, the demonstration of which requires us to consider systematically how people *act* on conclusions of conditional form which they might arrive at, and how the wanted or unwanted results of these actions are related to the ‘rightness’ of the conclusions acted on. I should perhaps acknowledge

immediately that the adequacy argument is anything but definitive, and perhaps the strongest claim that can be made for the significance of these arguments is that these are the sorts of considerations which *ought* to be taken into account in evaluating any proposed logical theory whose basic assumptions are questionable.

Chapter IV concerns counterfactuals, and covers much the same ground as another article 'Prior Probabilities and Counterfactual Conditionals' [5] which I had originally expected to appear prior to the book, but which will now be rendered obsolete by the book because of important modifications of the theory. The core of both the article and the book is an *epistemic past tense* hypothesis concerning the analysis of the counterfactual, according to which the probability of a counterfactual conditional at the time of its utterance equals a *prior probability* of the corresponding indicative conditional (i.e., its probability upon some prior occasion). This is argued to explain a variety of logical phenomena involving the counterfactual (possibly the most interesting of which is its use in 'explanation' contexts, where it clearly does *not* imply the falsity of its antecedent), and to yield a deeper understanding of inference 'processes' like a typical kind of *Modus Tollens*, in which 'inferring a conclusion' is reconceptualized as a phenomenon of *probability change* resulting from new premise acquisition. The chief difference between the present chapter and the article is that I no longer maintain that the epistemic past tense interpretation can be stretched to cover all uses of the counterfactual, and there are significant uses, especially related to *dispositional* concepts, which do not conform to the analysis. In consequence, I would now argue only that something like the epistemic past interpretation should play an important *part* in an adequate general analysis of the counterfactual, but lacking such an analysis it may be useful to consider the implications of the limited hypothesis.

A word should be said about the mathematical background presupposed of readers of this book. I should like to think that the proposed theory would be of interest to logicians generally, and I have accordingly kept mathematical technicalities to a minimum consistent with a reasonable demand for brevity. As noted, the only original and even slightly difficult mathematics is confined to Chapter II, and nearly all parts of the other chapters can be read independently. Elementary probability formulas are occasionally employed, which should be intelligible to

persons with only a small acquaintance with the formalism of probability, and occasionally some slight mathematical argument, which will be obvious to anyone knowing something of probability theory, is needed to justify these formulas, but which is omitted in order to avoid obscuring the fundamental issues at stake. Above all I have tried to avoid the appearance of mathematical display for its own sake, since I am most anxious that this work not be dismissed as just another of the puerile mathematical exercises in logical ‘system building’ which have become only too common in recent years (realistically, I must suppose that the book will be dismissed in this way by many).

It is impossible to acknowledge my indebtedness individually to all of the many students and colleagues whose ideas and criticisms have helped to shape my own ideas on conditionals, but I am particularly grateful to Professors Brian Skyrms, William Cooper, and David Shwayder. Skyrms has most influenced my thinking about counterfactuals, has gone much further than I have in investigating relations between counterfactuals and *laws*, and a book which he is now engaged in writing on that subject will very much deepen the rather sketchy ideas presented here. Cooper’s work has been most helpful to me in its systematic analysis of all sorts of uses of indicative conditionals in ordinary speech and writing, and again a forthcoming book [12] will go much further than I have in describing the ordinary language ‘data’ which any theory of the conditional must account for. Shwayder is the person who first suggested to me that I try to draw my ideas concerning conditionals together and present them in book form, and his influence here is most evident in the earlier sections of Chapter III, concerning the relation between belief and action. He also very kindly provided me with detailed criticisms and comments on those sections, which show that some aspects of the theory presented there require serious modification. Unfortunately, pressure to meet a publishing deadline has prevented me from taking these criticisms into account here, and all I can say here is that I intend to more fittingly acknowledge them in later work in the way that matters most: namely by demonstrating their influence on my own views.

I should like also to express my thanks to Savannah Ross, Yulia Motofuji, Julie Martinson, and Katy Dreith who have wrestled with the manuscript of this book in various stages of its preparation, and especially to Ruth Suzuki whose excellent and rapid typing alone allowed the manu-

script to be completed in a last mad scramble to finish in time to meet the publisher's deadline. Finally, a grateful salutation is in order to the persons to whom this book is dedicated, my wife Anne, and sons Jimmy and Billy, who have lived with and borne with the writing of this book almost as much as I have, and who I hope most of all will be pleased with the result.