

This is a sequel to the author's *Integral Calculus*. The notion of integration is extended to functions of several variables based on the intuitive idea of summation, but explicit treatment is limited to curvilinear, double, triple and surface integrals. Techniques are presented for evaluating those integrals, particularly those which occur in Physics.

Numerous examples are worked in the text and exercises, with solutions, are appended to each chapter.

MULTIPLE INTEGRALS

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Preface

The aim of this book is to give an elementary treatment of multiple integrals. The notions of integrals extended over a curve, a plane region, a surface and a solid are introduced in turn, and methods for evaluating these integrals are presented in detail. Especial reference is made to the results required in Physics and other mathematical sciences, in which multiple integrals are an indispensable tool.

A full theoretical discussion of this topic would involve deep problems of analysis and topology, which are outside the scope of this volume, and concessions had to be made in respect of completeness without, it is hoped, impairing precision and a reasonable standard of rigour. As in the author's *Integral Calculus* (in this series), the main existence theorems are first explained informally and then stated exactly, but not proved. Topological difficulties are circumvented by imposing somewhat stringent, though no unrealistic, restrictions on the regions of integration.

Numerous examples are worked out in the text, and each chapter is followed by a set of exercises.

My thanks are due to my colleague Dr. S. Swierczkowski, who read the manuscript and made valuable suggestions.

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