Hilbert Spaces, Wavelets, Generalised Functions and Modern Quantum Mechanics

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Hilbert Spaces, Wavelets, Generalised Functions and Modern Quantum Mechanics

by

Willi-Hans Steeb

International School for Scientific Computing,
Rand Afrikaans University,
Johannesburg, South Africa

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List of Symbols

$\emptyset$	empty set
$\mathbf{N}$	natural numbers
$\mathbf{Z}$	integers
$\mathbf{Q}$	rational numbers
$\mathbf{R}$	real numbers
$\mathbf{R}^+$	nonnegative real numbers
$\mathbf{C}$	complex numbers
$\mathbf{R}^n$	n-dimensional Euclidian space
$\mathbf{C}^n$	n-dimensional complex linear space
$\mathcal{H}$	Hilbert space
i	$:= \sqrt{-1}$
$\Re z$	real part of the complex number z
$\Im z$	imaginary part of the complex number z
$A \subset B$	subset A of set B
$A \cap B$	the intersection of the sets A and B
$A \cup B$	the union of the sets A and B
$f \circ g$	composition of two mappings $(f \circ g)(x) = f(g(x))$
ψ, $\|\psi\rangle$	wave function
t	independent variable (time variable)
x	independent variable (space variable)
$\mathbf{x} \in \mathbf{R}^n$	element $\mathbf{x}$ of $\mathbf{R}^n$
$\|\cdot\|$	norm
$\mathbf{x} \times \mathbf{y}$	vector product
$\otimes$	Kronecker product, tensor product
$\wedge$	exterior product (Grassmann product, wedge product)
$\langle\,,\rangle$, $\langle\,\|\,\rangle$	scalar product (inner product)
det	determinant of a square matrix
tr	trace of a square matrix
$\{\,,\}$	Poisson product
$[\,,]$	commutator
$[\,,]_+$	anticommutator
δ_{jk}	Kronecker delta
δ	delta function
$\mathrm{sgn}(x)$	the sign of x, 1 if $x > 0$, -1 if $x < 0$, 0 if $x = 0$
λ	eigenvalue
ϵ	real parameter

I	unit operator, unit matrix
U	unitary operator, unitary matrix
Π	projection operator, projection matrix
H	Hamilton function
$\hat{H}$	Hamilton operator
V	potential
b_j, $b_j^\dagger$	Bose operators
c_j, $c_j^\dagger$	Fermi operators
$\mathbf{p}$	momentum
$\hat{\mathbf{p}}$	momentum operator
$\mathbf{L}$	angular momentum
$\hat{\mathbf{L}}$	angular momentum operator
$\|\beta\rangle$	Bose coherent state
D	differential operator $\partial/\partial x$
Ω_+	Møller operator
$Y_{lm}(\theta, \phi)$	spherical harmonics

Preface

This book provides an introduction to Hilbert space theory, Fourier transform and wavelets, linear operators, generalized functions and quantum mechanics. Although quantum mechanics has been developed between 1925 and 1930 in the last twenty years a large number of new aspect and techniques have been introduced. The book also covers these new fields in quantum mechanics.

In quantum mechanics the basic mathematical tools are the theory of Hilbert spaces, the theory of linear operators, the theory of generalized functions and Lebesgue integration theory. Many excellent textbooks have been written on Hilbert space theory and linear operators in Hilbert spaces. Comprehensive surveys of this subject are given by Weidmann [68], Prugovečki [47], Yosida [69], Kato [31], Richtmyer [49], Sewell [54] and others. The theory of generalized functions is also well covered in good textbooks (Gelfand and Shilov [25], Vladimirov [67]. Furthermore numerous textbooks on quantum mechanics exist (Dirac [17], Landau and Lifshitz [36], Messiah [41], Gasiorowicz [24], Schiff [51], Eder [18] and others). Besides these books there are several problem books on quantum mechanics (Flügge [22], Constantinescu and Magyari [15], ter Haar [64], Mavromatis [39], Steeb [59], Steeb [60], Steeb [61]) and others). Computer algebra implementations of quantum mechanical problems are described by Steeb [59].

Unfortunately, many standard textbooks on quantum mechanics neglect the mathematical background. The basic mathematical tools to understand quantum mechanics should be fully integrated into an education in quantum mechanics.

The first four chapters of this book give an introduction to the mathematical tools necessary in quantum mechanics. The remaining chapters are devoted to quantum mechanics. The final chapter gives an introduction to Lebesgue integration theory.

The book covers new fields in quantum mechanics, such as coherent states, squeezed states, solitons and quantum mechanics, secular terms, Kronecker product and spin systems, and Berry phase, perturbation theory and differential equations, quantum measurement and quantum computing. These fields are not included in many standard textbooks in quantum mechanics.

Basic knowledge in linear algebra and calculus is required. It is also desirable for the reader to have basic knowledge in Hamilton mechanics. In almost all chapters a large number of examples serve to illustrate the mathematical tools. Most of the chapters include several exercises. A large number of references are given for further reading.

Ends of proofs are indicated by ♠. Ends of examples are indicated by ♣.

Any useful suggestions and comments are welcome. The e-mail address of the author is:

WHS@RAU3.RAU.AC.ZA

The web page of the author is:

http://zeus.rau.ac.za/steeb/steeb/html

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