

Superanalysis

Mathematics and Its Applications

Managing Editor:

M. HAZEWINKEL

Centre for Mathematics and Computer Science, Amsterdam, The Netherlands

Volume 470

Superanalysis

by

Andrei Khrennikov

*Department of Mathematics, Statistics and Computer Sciences,
University of Växjö,*

Växjö, Sweden

and

*Department of Mathematics,
Moscow State University of Electronic Engineering,
Zelenograd, Moscow, Russia*



SPRINGER SCIENCE+BUSINESS MEDIA, B.V.

A C.I.P. Catalogue record for this book is available from the Library of Congress.

ISBN 978-94-010-5948-0 ISBN 978-94-011-4609-8 (eBook)
DOI 10.1007/978-94-011-4609-8

Printed on acid-free paper

This is a completely updated and revised translation of the original Russian work of the same title.

Nauka, Moscow ©1997

All Rights Reserved

© 1999 Springer Science+Business Media Dordrecht

Originally published by Kluwer Academic Publishers in 1999

Softcover reprint of the hardcover 1st edition 1999

No part of the material protected by this copyright notice may be reproduced or utilized in any form or by any means, electronic or mechanical, including photocopying, recording or by any information storage and retrieval system, without written permission from the copyright owner

This book is dedicated to Professor Vasilii Vladimirov.

Table of Contents

Introduction	1
I Analysis on a Superspace over Banach Superalgebras	7
1. Differential Calculus	7
2. Cauchy–Riemann Conditions and the Condition of Λ -Linearity of Derivatives	21
3. Integral Calculus	26
4. Integration of Differential Forms of Commuting Variables	43
5. Review of the Development of Superanalysis	51
6. Unsolved Problems and Possible Generalizations	56
II Generalized Functions on a Superspace	57
1. Locally Convex Superalgebras and Supermodules	60
2. Analytic Generalized Functions on the Vladimirov–Volvich Superspace	63
3. Fourier Transformation of Superanalytic Generalized Functions	74
4. Superanalog of the Theory of Schwartz Distributions	92
5. Theorem of Existence of a Fundamental Solution	100
6. Unsolved Problems and Possible Generalizations	106

III	Distribution Theory on an Infinite-Dimensional Superspace	109
	1. Polylinear Algebra over Commutative Supermodules	110
	2. Banach Supermodules	116
	3. Hilbert Supermodules	130
	4. Duality of Topological Supermodules	141
	5. Differential Calculus on a Superspace over Topological Supermodules	144
	6. Analytic Distributions on a Superspace over Topological Supermodules	158
	7. Gaussian and Feynman Distributions	166
	8. Unsolved Problems and Possible Generalizations . . .	180
IV	Pseudodifferential Operators in Superanalysis	183
	1. Pseudodifferential Operators Calculus	183
	2. The Correspondence Principle	197
	3. The Feynman–Kac Formula for the Symbol of the Evolution Operator	205
	4. Unsolved Problems and Possible Generalizations . . .	221
V	Fundamentals of the Probability Theory on a Superspace	227
	1. Limit Theorems on a Superspace	227
	2. Random Processes on a Superspace	240
	3. Axiomatics of the Probability Theory over Superalgebras	244
	4. Unsolved Problems and Possible Generalizations . . .	254
VI	Non-Archimedean Superanalysis	257
	1. Differentiable and Analytic Functions	258
	2. Generalized Functions	264
	3. Laplace Transformation	267
	4. Gaussian Distributions	269
	5. Duhamel non-Archimedean Integral. Chronological Exponent	270

6. Cauchy Problem for Partial Differential Equations with Variable Coefficients	273
7. Non-Archimedean Supersymmetrical Quantum Mechanics	276
8. Trotter Formula for non-Archimedean Banach Algebras	278
9. Volkenborn Distribution on a non-Archimedean Superspace	279
10. Infinite-Dimensional non-Archimedean Superanalysis	283
11. Unsolved Problems and Possible Generalizations . . .	289
VII Noncommutative Analysis	293
1. Differential Calculus on a Superspace over a Noncommutative Banach Algebra	294
2. Differential Calculus on Noncommutative Banach Algebras and Modules	298
3. Generalized Functions of Noncommuting Variables .	309
VIII Applications in Physics	313
1. Quantization in Hilbert Supermodules	314
2. Transition Amplitudes and Distributions on the Space of Schwinger Sources	315
References	329
Index	345