

# Recent Developments in Quantum Mechanics

# MATHEMATICAL PHYSICS STUDIES

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# Recent Developments in Quantum Mechanics

*Proceedings of the Brasov Conference,  
Poiana Brasov 1989, Romania*

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## P R E F A C E

This book contains the invited lectures given at the 1989 Poiana Braşov Summer School on: "Recent Developments in Quantum Mechanics". The main aim of the School has been to present some expository talks reviewing the state of the art in some mathematical problems of Quantum Mechanics. A special emphasize was on the behaviour of Bloch electrons in electric and magnetic fields. In this context various techniques from semiclassical analysis and pseudodifferential operators are discussed. The volume also contains the "mini-course" on  $\mathcal{D}$ -modules and pseudodifferential operators given by prof. Louis Boutet de Monvel. Many other subjects of current interest in the Mathematical problems of Q.M. are also contained, as: stochastic methods, scattering theory, Supersymmetric Quantum Mechanics, etc.

We owe special thanks to all the invited lecturers for their contributions to the success of the School.

The School has been organized in the frame of the collaboration programme between the University of Paris VII and the Romanian Academy. We also thank UNESCO, CNRS and IAP for their financial help.

Many of our colleagues from the Institute of Atomic Physics helped us in various ways, and we thank them for that. We are especially grateful to Drs. V.Ceaşescu and V.Georgescu for their inestimable help.

Ms. Geta Uglai, the secretary of the School, handled the tedious work related to the School with high competence and efficiency, for which we owe her our warm gratitude.

The Editors

Bucharest

Ils  
pendent

à  
la verticale

Ils  
écoutent

le sable  
tomber

Ils  
entendent

la nuit  
s'écouler

Ils  
attendent

le jour  
pour chanter

LA VIE

et tout se termine  
au fond du sablier



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