Boundary Element Methods in Heat Transfer
International Series on Computational Engineering

Aims:
Computational Engineering has grown in power and diversity in recent years, and for the engineering community the advances are matched by their wider accessibility through modern workstations. The aim of this series is to provide a clear account of computational methods in engineering analysis and design, dealing with both established methods as well as those currently in a state of rapid development.

The series will cover books on the state-of-the-art development in computational engineering and as such will comprise several volumes every year covering the latest developments in the application of the methods to different engineering topics. Each volume will consist of authored work or edited volumes of several chapters written by the leading researchers in the field. The aim will be to provide the fundamental concepts of advances in computational methods as well as outlining the algorithms required to implement the techniques in practical engineering analysis.


Series Editor:
Dr C.A. Brebbia
Wessex Institute of Technology
Computational Mechanics Institute
Ashurst Lodge
Ashurst
Southampton SO4 2AA
UK

Associate Editor:
Dr M.H. Aliabadi
Wessex Institute of Technology
Computational Mechanics Institute
Ashurst Lodge
Ashurst
Southampton SO4 2AA
UK

Editorial Board:
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Technische Universität Braunschweig
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Germany

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10043 Orbassano (TO)  
Italy

Professor N.G. Zamani  
University of Windsor  
Department of Mathematics and Statistics  
401 Sunset  
Windsor  
Ontario  
Canada N9B 3P4

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Boundary Element Methods in Heat Transfer

Editors:
L.C. Wrobel and C.A. Brebbia

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**Chapter 4 - Solving Nonlinear Heat Transfer Problems Using the Boundary Element Method**  
*R. Bialecki*

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**Chapter 5 - Coupled Conduction-Convection Problems**  
*L.C. Wrobel, D.B. DeFigueiredo*

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**Chapter 6 - Solving Coupled Problems Involving Conduction, Convection and Thermal Radiation**  
*A.J. Nowak*

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**Chapter 7 - Advanced Thermoelastic Analysis**  
*V. Sladek, J. Sladek*

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Heat transfer problems in industry are usually of a very complex nature, simultaneously involving different transfer modes such as conduction, convection, radiation and others. Because of that, very few problems can be solved analytically and one generally has to resort to numerical analysis.

The boundary element method is a numerical technique which has been receiving growing attention for solving heat transfer problems because of its unique ability to confine the discretization process to the boundaries of the problem region. This allows major reductions in the data preparation and computer effort necessary to solve complex industrial problems.

The purpose of this book is to present efficient algorithms used in conjunction with the boundary element method for the solution of steady and transient, linear and nonlinear heat transfer problems. It also aims to reflect research being carried out by several active groups around the world, and its chapters have accordingly been written by scientists working in renowned centres of excellence.

The first three chapters all deal with transient heat conduction using alternative boundary element formulations which require boundary discretization only. Chapter 1 presents the dual reciprocity technique which is attracting considerable interest because of its ability to transform domain integrals, resulting from effects such as internal heat generation, into equivalent boundary integrals. The technique is general and is applied in this chapter to steady and transient, linear and nonlinear problems. A more traditional approach using time-dependent fundamental solutions is described in chapter 2. Also included is a discussion on the treatment of some types of initial conditions and internal loadings by equivalent boundary integrals, and an efficient convolution-type time-marching scheme. The multiple reciprocity method is described in chapter 3. This method may be seen as an extension of Galerkin-vector techniques for non-harmonic loads, and can also be applied to transient problems.

Chapter 4 deals with nonlinear heat transfer problems. The types of nonlinearity discussed include those of material, boundary conditions, heat sources and moving boundaries. Several practical examples of application are presented, and areas pointed out where further research is still necessary.

Boundary element solutions to the convection-diffusion equation are the subject of chapter 5. The fundamental solution to the steady-state equation with constant coefficients is employed, and features such as transient effects and variable parameters are accounted for by using dual reciprocity approximations. Coupled problems are also discussed in chapter 6 which deals with heat transfer involving conduction, convection and radiation in enclosures. The formulation developed can be implemented into standard boundary element codes, and is equivalent to introducing a new fundamental solution. Special consideration is given to the resulting set of nonlinear equations which is solved by an efficient pre-elimination technique employing the Gauss-Jordan algorithm.

Chapter 7, on thermoelasticity, starts with a brief classification of thermoelastic problems; next, it defines the fundamental solution for the Laplace transforms in general coupled thermoelasticity, and the time-dependent fundamental solutions when
these are available. A pure boundary formulation is then given for both the Laplace transform and the time-dependent fields, and boundary integral equations written in an advanced regularized form without any singular integral. Finally, a BEM formulation for solution of stationary problems in media with temperature-dependent Young's modulus and coefficient of thermal expansion is presented.

Natural convection in fluid flow is the subject of chapter 8. This chapter is a collection of recent results obtained by the authors using an integral equation method based on boundary-domain discretization for solving two-dimensional thermal convection problems. The formulation uses the primitive variables, i.e. velocity and pressure, and constructs fundamental solution tensors for the differential operators corresponding to a linearized set of equations. Approximate solution procedures of the nonlinear system of integral equations are derived based on Newton-Raphson techniques.

The last chapter deals with inverse heat conduction problems. Three different mathematical models, namely direct, least squares and minimum energy methods, are presented for two Laplace-type problems. It is found that the minimum energy method always gives a good, stable approximation to the solution, whereas the direct and least squares methods do not.

We are indebted to all the authors for their contribution, patience and continuous support during the production stages of this book. Special thanks are due to Ms. Christine Seward for the excellent work in the preparation of the final manuscript.

Luiz C. Wrobel
Carlos A. Brebbia

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