Hamiltonian Mechanical Systems and Geometric Quantization

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# Hamiltonian Mechanical Systems and Geometric Quantization

*by*

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# Introduction

The book is a revised and updated version of the lectures given by the author at the University of Timişoara during the academic year 1990–1991. Its goal is to present in detail some old and new aspects of the geometry of symplectic and Poisson manifolds and to point out some of their applications in Hamiltonian mechanics and geometric quantization.

The material is organized as follows. In Chapter 1 we collect some general facts about symplectic vector spaces, symplectic manifolds and symplectic reduction. Chapter 2 deals with the study of Hamiltonian mechanics. We present here the general theory of Hamiltonian mechanical systems, the theory of the corresponding Poisson bracket and also some examples of infinite–dimensional Hamiltonian mechanical systems. Chapter 3 starts with some standard facts concerning the theory of Lie groups and Lie algebras and then continues with the theory of momentum mappings and the Marsden–Weinstein reduction. The theory of Hamilton–Poisson mechanical systems makes the object of Chapter 4. Chapter 5 is dedicated to the study of the stability of the equilibrium solutions of the Hamiltonian and the Hamilton–Poisson mechanical systems. We present here some of the remarcable results due to Holm, Marsden, Raţiu and Weinstein. Next, Chapter 6 and 7 are devoted to the theory of geometric quantization where we try to solve, in a geometrical way, the so called Dirac problem from quantum mechanics. We follow here the construction given by Kostant and Souriau around 1964. Foliated cohomology, the theory of the Dolbeault–Kostant complex and their applications in geometric quantization are presented in Chapter 8. In Chapter 9 we discuss the relation between geometric quantization and the Marsden–Weinstein reduction and point out some of its applications in the geometric quantization of a constrained mechanical system. Finally in the last chapter we try to extend the theory of geometric quantization to the Poisson manifolds via the theory of symplectic groupoids in the sense of Karasev and Weinstein.

At the end of each chapter there is a set of problems with solutions. Some problems are routine and test the general understanding of the chapters. Many present significant applications of the text and in some cases the problems contain major theorems.

I am happy to express my gratitude to Professors Jerrold Marsden, Tudor Raţiu and Alan Weinstein for their permanent support and encouragement. It was a privilege for me to be in permanent contact with them. I owe many thanks to my major Professor Dan I. Papuc. He guided me not only in mathematics but also in a lot of problems of life. Thanks are also due to my wife Liliana for her love, patience and kindness. Last but not least, I want to thank Kluwer Academic Publishers who have given me the opportunity to publish my book in their collection.

Timişoara
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Mircea Puta

# Background Notations

The reader is assumed to be familiar with the usual notations of set theory such as: $\in$, $\subset$, $\cup$, $\cap$. Other notations we shall use without explanation include the following:

| | |
|---|---|
| ● | end of an example or remark |
| ■ | end of a proof or a solution |
| iff | if and only if |
| $A \times B$ | cartesian product |
| $A \setminus B$ | set theoretic difference |
| $f^{-1}(B)$ | inverse image of $B$ under $f$ |
| Id | identity map |
| $\ker(t)$ | kernel of a linear map $t$ |
| $\mathrm{range}(t)$ | range of a linear map $t$ |
| $a_i b^i$ | summation over $i$ understood |
| $\delta_{ij} = \begin{cases} 1 & i = j \\ 0 & i \neq j \end{cases}$ | Kronecker index |
| smooth | smooth of class $C^\infty$ |
| R, C | real or complex numbers |
| Z, Q | integers, rational numbers |
| $R^n$, $C^n$ | Euclidean $n$–space, complex $n$–space |

Throughout this book we shall assume that the manifolds are finite–dimensional, smooth, paracompact, connected and without boundary. The exceptions will be mentioned always in the text.