Elliptically Contoured Models in Statistics

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Elliptically Contoured Models in Statistics

by

A. K. Gupta

Department of Mathematics and Statistics,
Bowling Green State University,
Bowling Green, Ohio, U.S.A.

and

T. Varga

AB-AEGON General Insurance Company,
Budapest, Hungary

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Dedicated to the memory of my mother and father. AKG

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PREFACE

In multivariate statistical analysis, elliptical distributions have recently provided an alternative to the normal model. Most of the work, however, is spread out in journals throughout the world and is not easily accessible to the investigators. Fang, Kotz, and Ng presented a systematic study of multivariate elliptical distributions, however, they did not discuss the matrix variate case. Recently Fang and Zhang have summarized the results of generalized multivariate analysis which include vector as well as the matrix variate distributions. On the other hand, Fang and Anderson collected research papers on matrix variate elliptical distributions, many of them published for the first time in English. They published very rich material on the topic, but the results are given in paper form which does not provide a unified treatment of the theory. Therefore, it seemed appropriate to collect the most important results on the theory of matrix variate elliptically contoured distributions available in the literature and organize them in a unified manner that can serve as an introduction to the subject.

The book will be useful for researchers, teachers, and graduate students in statistics and related fields whose interests involve multivariate statistical analysis. Parts of this book were presented by Arjun K.Gupta as a one semester course at Bowling Green State University. Some new results have also been included which generalize the results in Fang and Zhang. Knowledge of matrix algebra and statistics at the level of Anderson is assumed. However, Chapter 1 summarizes some results of matrix algebra. This chapter also contains a brief review of the literature and a list of mathematical symbols used in the book.

Chapter 2 gives the basic properties of the matrix variate elliptically contoured distributions, such as the probability density function and expected values. It also presents one of the most important tools of the theory of elliptical distributions, the stochastic representation.

The probability density function and expected values are investigated in detail in Chapter 3.

Chapter 4 focuses on elliptically contoured distributions that can be represented as mixtures of normal distributions.

The distributions of functions of random matrices with elliptically contoured distributions are discussed in Chapter 5. Special attention is given to quadratic forms.

Characterization results are given in Chapter 6.

The last three chapters are devoted to statistical inference. Chapter 7 focuses on estimation results, whereas Chapter 8 is concerned with hypothesis testing problems. Inference for linear models is studied in Chapter 9. Finally, an up to date bibliography has been provided, along with author and subject indexes.

We would like to thank the Department of Mathematics and Statistics, Bowling Green State University, for supporting our endeavour and for providing the necessary facilities to accomplish the task. The first author is thankful to the Biostatistics Department, University of Michigan, for providing him the opportunity to organize the material in its final form. Thanks are also due to Professors A. M. Kshirsagar, D. K. Nagar, M. Siotani, and J. Tang for many helpful discussions. He would also like to acknowledge his wife, Meera, and his children, Alka, Mita, and Nisha for their support throughout the writing of the book.

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A. K. Gupta
T. Varga