

ALLOCATION MODELS AND THEIR USE IN ECONOMIC PLANNING

A. R. G. HEESTERMAN

ALLOCATION MODELS
AND THEIR USE
IN ECONOMIC PLANNING



D. REIDEL PUBLISHING COMPANY / DORDRECHT-HOLLAND

ISBN-13 978-94-010-3086-1

e-ISBN-13: 978-94-010-3084-7

DOI: 10.100/978-94-010-3084-7

Library of Congress Catalog Card Number 78-146964

All Rights Reserved

Copyright © 1971 by D. Reidel Publishing Company, Dordrecht, Holland

Softcover reprint of the hardcover 1st edition 1971

**No part of this book may be reproduced in any form, by print, photoprint, microfilm,
or any other means, without written permission from the publisher**

ACKNOWLEDGEMENTS

When seeing this manuscript off to the publisher, I should mention my indebtedness to the following people:

First of all, in a general way, the people in those low-income countries who, trying to come out of this category, did not have any marked success so far. It was my concern for this problem which focussed my attention on the problems of economic planning, discussed in this book. In retrospect, I mention this point with somewhat mixed feelings, since so far, the tangible benefits are for me.

My second acknowledgement should go to the University's National Economic Planning Unit, for asking me to teach this particular subject, otherwise I would probably not have come to the stage of writing a book on it.

The next acknowledgement goes to my students, for their participation in discussion, both during and out of lecture-hours, thereby clarifying my own ideas on a number of points, as well as for noticing errors in the manuscript in its preliminary stage of lecture-handout, and for asking clarification on certain points, thereby prompting clarification extension of the book-version as well.

Finally, I acknowledge the competent typing of most of the manuscript by Mrs. Lyn Fowles, my secretary at the time, and of Mrs. Wiebien Heesterman and Mrs. Hazel Shaul, who screened the manuscript for errors in the English language, although I was obstinate enough not always to follow the suggested alterations.

A. R. G. HEESTERMAN

*University of Birmingham, U.K.
22nd September, 1970*

SUMMARY AND INTRODUCTION

Three different lines of approach have contributed to the theory of optimal planning. One approach considers the problem from the view-point of a national government and its adviser, the econometrician planning specialist. The government can, if this is thought to be desirable, stimulate investment in certain directions and discourage other economic activities. By various fiscal devices, it can influence both the total level and the distribution of investment funds over different sectors of production.

Also, in many countries, a public agency plays some kind of coordinating role in the formulation of long-term plans for output by the enterprises sector; this may range from administrative direction in so-called centrally planned economies, to persuasion and advice in 'capitalist' economies. Accordingly, the public planner wishes to know what distribution of the nation's resources would be 'optimal'. This leads to the construction of various models which may be described under the general heading 'input-output type models'.

This type of model has been largely developed by practitioners, among whom Sandee [B2] is probably the most outstanding and the earliest. A later, well-developed example of a model based on this approach is, for example, the Czech model by Černý *et al.* [B1]. A second approach considers the problem from the point of view of the private entrepreneur and his adviser, the manager and financial accountant.

While the first approach talks in terms of millions of man-years, export ceilings and annual percentages of growth for different sectors, the second approach talks in terms of costs and profitability, and rates of return on investment. The second approach has a slightly more established tradition, so that textbook manuals¹ are now available.

The next approach is welfare economics. The subject of our discourse is strictly within the province of welfare economics. However, it will be argued in this book that prices and costs are determinate outside the institutional framework of a financial economy only if we make certain restrictive assumptions, the nature of which is not widely appreciated. By

SUMMARY AND INTRODUCTION

implication, prices and costs, as actually paid, are indeterminate except for the influence of political factors, unless the same assumptions are upheld. From a point of view of logic, it is not at all certain whether we may assume a unique set of efficiency prices. The truth is that we make these assumptions because we *need* calculation prices.

Between the different approaches to optimality there is still a gap. First of all, there is a considerable 'language barrier'. The 'planning' approach uses linear programming models with supposedly real statistical data; welfare economics and economic analysis speak in terms of calculus, first order derivatives or Lagrange multipliers or use linguistic terms which imply these mathematical concepts; and the practitioners of the actual investment decisions use no higher mathematics at all, unless one wishes to consider the computation of compound interest as such. There are also still some differences in the substance between the three approaches. This book is an attempt to bridge the still remaining gaps, and as far as possible, to break down the language barrier either by speaking several 'languages' or by using terms that are comprehensible to the protagonists of all three approaches.

The book consists of three parts. Part I on efficiency prices is chiefly concerned with the reconciliation of the planning and programming approach and the economic analysis approach. To this purpose, I first of all formulated a standard framework for the planning and programming approach 'the generalized input-output model'. All exogenous restrictions, like the limited availability of land and labour, but also upper limits to exports, and even such restrictions as limits on social mobility, can be thought of as production factors. All variable activities may be classified as production processes. For land and labour as production factors and metal products as a good, this is a familiar notion. But we may also think of export as a production process which transfers a certain good into foreign exchange by exporting it. An export limit, due to the absorption limit of the outside markets, will then become a production factor limit, just like the supply limit on labour or land; and a 'rent', a factor income may arise from it. The generalized input-output model is then scrutinized by means of optimality conditions.

In this connection the concept of efficiency prices arises. This involves some bending of what was originally the planning and programming approach, but this is necessary in order to reach the cost accounting point

SUMMARY AND INTRODUCTION

of view. If two hospitals *or* two tenths of a percent more consumption are possible alternatives (feasible solutions to the model), the programming approach considers it possible that one could have one hospital and one tenth of a percent more consumption, *plus* a part of yet another hospital, the model being convex. But the efficiency prices concept assumes a normally *flat* transformation surface, *i.e.* one can have one hospital and one tenth of a percent more consumption *and nothing more*.

Efficiency prices may be considered as dual variables, associated with the programming model. They lead to results which can be interpreted in economic terms, thereby leading to a frame of reference which is familiar to the accountant. Conversely, it then becomes possible to use accounting methods in order to test for efficiency. The only optimality conditions used are the Kuhn-Tucker conditions [E6]. A treatise on optimality conditions is given in the appendix.

The second part of the book considers the evaluation of individual investment projects. The dynamic equivalent of zero profits is zero project value, measured by Discounted Cash Flow. The 'cash flows', however, need not necessarily *be* the actual cash flows. One has to assume that the prices used are efficiency prices, and these may be different from actual exchange prices. Some excursions into problems of indivisibilities and economics of scale are also included in this part of the book. Although not being an expert in capital budgeting, but rather an econometrician interested in economic planning, I felt that the no-mans land between econometrics and capital budgeting could not properly be explored without actually moving into the other territory.

Part III is concerned with a number of direct practical problems with which the planner may be confronted. The frame of reference is efficiency prices and the adjustment of exchange prices in order to obtain a price structure that might conceivably satisfy the conditions for efficiency prices. Computational methods of price-adjustment are discussed in Parts I and II; Part III concentrates on the classification of social and political reasons why exchange-prices are different from efficiency-prices.

A more theoretical section on the functions of prices is included in this part. The purpose of this section, nor of part III as a whole, is not, however, to discuss price-formation in the real world, but to discuss the reasons for price-adjustment, when costing projects.

The capita selecta in this part assume that the planner tries to steer by

SUMMARY AND INTRODUCTION

the compass of exchange-prices, and indicates the conditions when imperfection of this compass may be inferred from the theory of allocation models, even if no computed model is available. They are so to say, elements of a theory of cost-accounting of a planned economy.

NOTE TO SUMMARY AND INTRODUCTION

¹ See Edge, C. [C2] or more elaborate:
Bierman, H. and Smit, S. [C1].

TABLE OF CONTENTS

ACKNOWLEDGEMENTS	V
SUMMARY AND INTRODUCTION	VII
PART I. ALLOCATION, INVESTMENT AND EFFICIENCY PRICES IN INPUT-OUTPUT TYPE MODELS	
CHAPTER I. WHAT IS EFFICIENCY?	
1.1. The Preference Function	3
1.2. The Efficiency Frontier	5
1.3. Dynamic Efficiency	6
1.4. Efficiency Prices and Limiting Prices	7
CHAPTER II. THE GENERALIZED INPUT-OUTPUT MODEL	
2.1. The Primal of the Static Model	9
2.2. The Zero Profit Requirement	13
2.3. The Maximization of G.D.P.	14
2.4. Public versus Private Preferences	15
2.5. Arrow's Theorem on the Choice of Processes	16
2.6. Factor Substitution and Output Composition	21
2.7. The (Non) uniqueness of the Price Structure	27
2.8. The Adjusted Plan	34
2.9. The Plan-Orientated Preference Function	39
2.10. Price Adjustment in Input-Output Type Models	47
2.11. External Economies and Non-Convexity	54
2.12. Capacity in Transportation Problems	62
CHAPTER III. INTER-TEMPORAL ALLOCATION IN THE GENERALIZED MODEL	
3.1. The Multi-Period Model	67

TABLE OF CONTENTS

3.2. The Principle of Discount	70
3.3. The Recursive Formulation	72
3.4. An Example of a Multi-Period Allocation Model	74
3.5. The Dynamized Dual Restrictions	76
3.6. Interest Discount and Depreciation	78
3.7. Technical Change	79
3.8. The Plan-Orientated Intertemporal Preference Function	81
3.9. Some Questions for the Student	90
CHAPTER IV. THE BALANCED GROWTH FRONTIER	
4.1. The Exogenous Rate of Growth	92
4.2. A Demonstration Example of a Balanced Growth Path	93
4.3. The (Balanced Growth) Rate of Interest	99
4.4. The Balanced Growth Transformation Ridge	100
CHAPTER V. THE DYNAMIZED LEONTIEF MODEL	
5.1. Formal Specification of the Model	105
5.2. The Turnpike Rate of Growth	106
5.3. The Turnpike rate of Interest	110
5.4. The Dynamic Input-Output Model and Its Limiting Prices	112
CHAPTER VI. FOREIGN TRADE IN THE NATIONAL ECONOMY MODEL	
6.1. The Accounting Framework	116
6.2. Export and Market Limits	118
6.3. The Dynamic One-Factor Model with Foreign Trade	120
PART II. THE EVALUATION OF INDIVIDUAL PROJECTS	
CHAPTER VII. THE COSTING PROBLEM	
7.1. The Investment Decision	127
7.2. The Econometrician and the Accountant	129
7.3. A Short Summary of Methods of Investment Evaluation	132

TABLE OF CONTENTS

7.4. Project Evaluation and the Zero Profit Requirement	133
7.5. Natural Limits to Projects	133
CHAPTER VIII. DISCOUNTED CASH FLOW IN THE STANDARD CASE	
8.1. Present Value	136
8.2. How to Find the Rate of Interest	140
8.3. Input-Output Plan and Price Adjustment	143
8.4. Sector-Wise Discount	146
8.5. Clusters of Projects	148
8.6. Direct Coordination of Projects	150
8.7. Summary of a Planning Procedure	153
8.8. Some Questions for the Student	154
CHAPTER IX. INCREASING RETURNS TO SCALE	
9.1. Discussion of the Problem	156
9.2. Planned Surplus Capacity	158
9.3. Project Size and Efficiency Price	160
CHAPTER X. SOME SPECIAL EVALUATION PROBLEMS IN PARTICULAR SECTORS	
10.1. Investment in Transport Production	162
10.2. Education	163
10.3. Hospitals	164
PART III. CAPITA SELECTA ON ECONOMIC POLICY	
CHAPTER XI. THE DISTRIBUTION OF OUTPUTS	
11.1. The Functions of Prices	167
11.2. Profits	168
11.3. Duties on Final Outputs	170
11.4. Revenue Taxes	171

TABLE OF CONTENTS

11.5. Redistributive Taxation	173
11.6. Rationing	173
CHAPTER XII. OPPORTUNITY COST AND EXCHANGE PRICE	
12.1. Discussion of the Problem	176
12.2. Costing versus Programming	176
12.3. Macro-economic Equilibrium	178
12.4. Technical Change	179
12.5. Import Substitution and the Cost of Foreign Exchange	180
12.6. Underutilization of Resources	181
APPENDIX A. OPTIMALITY CONDITIONS	
a.1. The Additive Property of Inequalities	184
a.2. The Programming Problem and Its Lagrangean	185
a.3. John's Theorem	186
a.4. Aggregate Restrictions of Allocation Models	192
a.5. The Kuhn-Tucker Theorem for Convex Programming	193
APPENDIX B. SOME CONVENTIONS OF NOTATION	198
BIBLIOGRAPHY	201