INVESTIGATIONS IN MODAL AND TENSE LOGICS
WITH APPLICATIONS TO PROBLEMS
IN PHILOSOPHY AND LINGUISTICS
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SOCIOLOGY OF SCIENCE AND OF KNOWLEDGE,
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VOLUME 92
To the Memory of my Father
Murad H. Gabbay
To my Wife
Lydia Rivlin
and
To my Teacher and Friend
Azriel Levy
For Everything They Have Done For Me
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This book is intended to serve as an advanced text and reference work on modal logic, a subject of growing importance which has applications to philosophy and linguistics. Although it is based mainly on research which I carried out during the years 1969–1973, it also includes some related results obtained by other workers in the field (see the references in Part 7).

Parts 0, 1 and 2, can be used as the basis of a one year graduate course in modal logic. The material which they contain has been taught in such courses at Stanford since 1970. The remaining parts of the book contain more than enough material for a second course in modal logic. The exercises supplement the text and are usually difficult.

I wish to thank Stanford University and Bar-Ilan University for making it possible for me to continue and finish this work, and A. Ungar for correcting the typescript.

Bar-Ilan University, Israel

DOV M. GABBAY