

# Maximum-Entropy and Bayesian Methods in Science and Engineering

Volume 1: Foundations

# Fundamental Theories of Physics

*An International Book Series on The Fundamental Theories of Physics: Their Clarification, Development and Application*

**Editor:** ALWYN VAN DER MERWE

*University of Denver, U.S.A.*

## **Editorial Advisory Board:**

ASIM BARUT, *University of Colorado, U.S.A.*

HERMANN BONDI, *University of Cambridge, U.K.*

BRIAN D. JOSEPHSON, *University of Cambridge, U.K.*

CLIVE KILMISTER, *University of London, U.K.*

GÜNTER LUDWIG, *Philipps-Universität, Marburg, F.R.G.*

NATHAN ROSEN, *Israel Institute of Technology, Israel*

MENDEL SACHS, *State University of New York at Buffalo, U.S.A.*

ABDUS SALAM, *International Centre for Theoretical Physics, Trieste,  
Italy*

HANS-JÜRGEN TREDER, *Zentralinstitut für Astrophysik der  
Akademie der Wissenschaften, G.D.R.*

# Maximum-Entropy and Bayesian Methods in Science and Engineering

Volume 1: Foundations

*edited by*

Gary J. Erickson

*Department of Electrical Engineering,  
Seattle University, Seattle, Washington, U.S.A.*

and

C. Ray Smith

*Advanced Sensors Directorate  
Research, Development and Engineering Center,  
US Army Missile Command, Redstone Arsenal,  
Alabama, U.S.A.*



**KLUWER ACADEMIC PUBLISHERS**

DORDRECHT / BOSTON / LONDON

Library of Congress Cataloging in Publication Data

ISBN 978-94-010-7871-9

ISBN 978-94-009-3049-0 (eBook)

DOI 10.1007/978-94-009-3049-0

---

Published by Kluwer Academic Publishers,  
P.O. Box 17, 3300 AA Dordrecht, The Netherlands.

Kluwer Academic Publishers incorporates  
the publishing programmes of  
D. Reidel, Martinus Nijhoff, Dr W. Junk and MTP Press.

Sold and distributed in the U.S.A. and Canada  
by Kluwer Academic Publishers,  
101 Philip Drive, Norwell, MA 02061, U.S.A.

In all other countries, sold and distributed  
by Kluwer Academic Publishers Group,  
P.O. Box 322, 3300 AH Dordrecht, The Netherlands.

All Rights Reserved

© 1988 by Kluwer Academic Publishers

**Softcover reprint of the hardcover 1st edition 1988**

No part of the material protected by this copyright notice may be reproduced or  
utilized in any form or by any means, electronic or mechanical  
including photocopying, recording or by any information storage and  
retrieval system, without written permission from the copyright owner.

*In honour of E. T. Jaynes*

*Of Related Interest*

# **Maximum-Entropy and Bayesian Methods in Inverse Problems**

Edited by

**C. Ray Smith and W. T. Grandy, Jr.**

*Department of Physics and Astronomy,  
The University of Wyoming, Laramie, Wyoming, U.S.A.*

This volume is the outcome of two workshops entitled "Maximum-Entropy and Bayesian Methods in Applied Statistics" and presents contributions by renowned authorities in many different fields. The purpose of these workshops was to bring together leading scientists whose research involved using the Principle of Maximum Entropy in a wide range of different applications in order to pool the experience gained and to identify common problems in need of solution. The result is stimulating and informative and provides many directions for further progress.

## *Audience*

"Maximum-Entropy and Bayesian Methods in Inverse Problems" will be of great interest to mathematicians, physicists, geophysicists, electrical engineers, economists, and those working in communication and information theory and many other aspects of signal processing.

*Of Related Interest*

# **Maximum-Entropy and Bayesian Spectral Analysis and Estimation Problems**

Edited by

**C. Ray Smith**

*U.S Army Missile Command, Redstone Arsenal, Alabama, U.S.A.*

and

**Gary J. Erickson**

*Department of Electrical Engineering,  
Seattle University, Seattle, Washington, U.S.A.*

This volume contains 20 contributions by leading researchers from different fields who critically examine maximum-entropy and Bayesian methods in science, engineering, signal processing, medical physics, and other disciplines.

This is a sequel to the volume 'Maximum-Entropy and Bayesian Methods in Inverse Problems', published by Reidel in 1985.

## *Audience*

This book will be of interest to probability theorists, statisticians, electrical engineers, communication engineers, computer scientists, physicists, mathematicians, biologists, geophysicists, and those working in medical imaging.

ISBN 978-94-010-7871-9

Volume 2: Applications

CONTENTS

PREFACE	ix
SETI, RADON TRANSFORMS, AND OPTIMIZATION Stanley R. Deans	1
INDUCTIVE INFERENCE BY NEUTRAL NETWORKS David Hestenes	19
ON THE EFFICIENCY OF A CLASS OF MAXIMUM ENTROPY ESTIMATORS C.C. Rodriguez	33
QUANTUM STATISTICAL MECHANICS IN PHASE SPACE AND THE CLASSICAL LIMIT Y. Tikochinsky and D. Shalitin	51
SUPERPOSITION EFFECTS IN DIFFERENTIAL ENTROPY AND KULLBACK- LEIBLER INFORMATION A.K. Rajagopal, P.J. Lin-Chung and S. Teitler	83
SUPER VARIATIONAL PRINCIPLES L.H. Schick	89
EINSTEIN'S REVERSAL OF THE BOLTZMANN PRINCIPLE AND PARTICLE STATISTICS A.K. Rajagopal and S. Teitler	105
CLASSICAL ENTROPY OF A COHERENT SPIN STATE: A LOCAL MINIMUM C.T. Lee	111
LEAST MAXIMUM ENTROPY AND MINIMUM UNCERTAINTY COHERENT STATES A.K. Rajagopal and S. Teitler	121
MAXIMUM ENTROPY SPECTROSCOPY - DIMES AND MESA John Skilling	127
INFORMATION AND ENTROPY OF PATTERNS IN GENETIC SWITCHES Thomas Dana Schneider	147
MAXIMUM ENTROPY AND THE PHASE PROBLEM IN PROTEIN CRYSTALLOGRAPHY R.K. Bryan	155



CONTRAST TRANSFER FUNCTION CORRECTION IN ELECTRON MICROSCOPY R.K. Bryan	171
CLIMATICALLY INDUCED CYCLIC VARIATIONS IN UNITED STATES CROP PRODUCTION: IMPLICATIONS IN ECONOMIC AND SOCIAL SCIENCE Robert Guinn Currie	181
A MAXIMUM ENTROPY METHOD FOR EXPERT SYSTEM CONSTRUCTION Alan Lippman	243
STOCHASTIC RELAXATION METHODS FOR IMAGE RESTORATION AND EXPERT SYSTEMS Stuart Geman	265
TOWARDS A METHOD OF CORRECTING BIAS INTRODUCED BY THE USE OF THE ERROR FITTING METHOD IN CONJUNCTION WITH A MAXIMUM ENTROPY PROCESSING ALGORITHM N.A. Farrow and F.P. Ottensmeyer	313
MAKING MAXIMUM ENTROPY COMPUTATIONS EASIER BY ADDING EXTRA CONSTRAINTS (EXTENDED ABSTRACT) Sally A. Goldman and Ronald L. Rivest	323
IMAGE RESTORATION AND RECONSTRUCTION USING ENTROPY AS A REGULARIZATION FUNCTIONAL Ali Mohammad-Djafari and Guy Demoment	341
APPLICATION OF LIKELIHOOD AND ENTROPY FOR TOEPLITZ CONSTRAINED COVARIANCE ESTIMATION Michael I. Miller	357
THE CONCEPT OF EPOCH ENTROPY IN COMPLEX SYSTEMS K.L. Ngai, A.K. Rajagopal and S. Teitler	363
A GENERAL THEORY OF INHOMOGENEOUS SYSTEMS S.A. Trugman	371
MAXIMUM ENTROPY AND CRACK GEOMETRY IN GRANITIC ROCKS P.M. Doyen	381
MAXIMUM ENTROPY ANALYSIS OF LIQUID DIFFRACTION DATA John H. Root, P.A. Egelstaff and B.G. Nickel	395
RANDOM ARRAY BEAMFORMING Keith H. Norsworthy and Paul N. Michels	409
DECISION MAKING WITH BARELY ANY INFORMATION: THE ROLE OF MIXED STRATEGIES P.B. Kantor and M.J. Kantor	421
COMPARISON OF BAYESIAN AND DEMPSTER'S RULES IN EVIDENCE COMBINATION Yizong Cheng and R.L. Kashyap	427
Subject Index	435

CONTENTS

PREFACE	ix
HOW DOES THE BRAIN DO PLAUSIBLE REASONING? E.T. Jaynes	1
THE RELATION OF BAYESIAN AND MAXIMUM ENTROPY METHODS E.T. Jaynes	25
AN ENGINEER LOOKS AT BAYES Myron Tribus	31
BAYESIAN INDUCTIVE INFERENCE AND MAXIMUM ENTROPY Stephen F. Gull	53
EXCERPTS FROM BAYESIAN SPECTRUM ANALYSIS AND PARAMETER ESTIMATION G. Larry Bretthorst	75
DETECTION OF EXTRA-SOLAR SYSTEM PLANETS E.T. Jaynes	147
STOCHASTIC COMPLEXITY AND THE MAXIMUM ENTROPY PRINCIPLE Jorma Rissanen	161
THE AXIOMS OF MAXIMUM ENTROPY John Skilling	173
UNDERSTANDING IGNORANCE C.C. Rodriguez	189
MAXIMUM ENTROPY CALCULATIONS ON A DISCRETE PROBABILITY SPACE P.F. Fougere	205
QUANTUM DENSITY MATRIX AND ENTROPIC UNCERTAINTY R. Blankenbecler and M.H. Partovi	235
INFORMATION-THEORETICAL GENERALIZATION OF THE UNCERTAINTY PRINCIPLE A.J.M. Garrett	245
TIME, ENERGY, AND THE LIMITS OF MEASUREMENT M.H. Partovi and R. Blankenbecler	249

ON A DETECTION ESTIMATOR RELATED TO ENTROPY R.N. Madan	257
THE EVOLUTION OF CARNOT'S PRINCIPLE E.T. Jaynes	267
A LOGIC OF INFORMATION SYSTEMS N.C. Dalkey	283
METHODOLOGICAL PRINCIPLES OF UNCERTAINTY IN INDUCTIVE MODELLING: A NEW PERSPECTIVE G.J. Klir	295
COMPARISON OF MINIMUM CROSS-ENTROPY INFERENCE WITH MINIMALLY INFORMATIVE INFORMATION SYSTEMS N.C. Dalkey	305
Subject Index	313

## PREFACE

This volume has its origin in the Fifth, Sixth and Seventh Workshops on "Maximum-Entropy and Bayesian Methods in Applied Statistics", held at the University of Wyoming, August 5-8, 1985, and at Seattle University, August 5-8, 1986, and August 4-7, 1987. It was anticipated that the proceedings of these workshops would be combined, so most of the papers were not collected until after the seventh workshop. Because all of the papers in this volume are on foundations, it is believed that the contents of this volume will be of lasting interest to the Bayesian community.

The workshop was organized to bring together researchers from different fields to critically examine maximum-entropy and Bayesian methods in science and engineering as well as other disciplines. Some of the papers were chosen specifically to kindle interest in new areas that may offer new tools or insight to the reader or to stimulate work on pressing problems that appear to be ideally suited to the maximum-entropy or Bayesian method. A few papers presented at the workshops are not included in these proceedings, but a number of additional papers not presented at the workshop are included. In particular, we are delighted to make available Professor E. T. Jaynes' unpublished Stanford University Microwave Laboratory Report No. 421 "How Does the Brain Do Plausible Reasoning?" (dated August 1957). This is a beautiful, detailed tutorial on the Cox-Polya-Jaynes approach to Bayesian probability theory and the maximum-entropy principle. In addition to the paper just described, we have included three more by Professor Jaynes: "The Relation of Bayesian and Maximum-Entropy Methods" (presented at the fifth workshop). "The Evolution of Carnot's Theory" (based upon a talk given at an EMBO Workshop in 1984) and "Detection of Extra-Solar System Planets" (made available for this volume). This last paper should pique the interest of anyone concerned with "superresolution". Incidentally, Professor Jaynes refers in this paper to the Ph.D. thesis of G. L. Bretthorst; much of Dr. Bretthorst's thesis appears as Chapter 5 of this volume.

These workshops and their proceedings could not have been brought to their final form without the support or help of a number of people. Professor Alwyn van der Merwe, the Editor of Fundamental Theories of Physics, and Dr. D. J. Larner of Kluwer, provided encouragement and friendship at critical times. Others who have made our work easier or more rewarding include Professor Paul D. Neudorfer of Seattle University, Mr. Robert M. Braukus, P.E., Director of Telecommunications of Puget Sound Power and Light Co., Dr. J. M. Loomis of the Radar Technology Branch of MICOM's Research, Development, and Engineering Center, and Dr. Rabinder Madan of the Office of Naval Research.

Partial support of the fifth and seventh workshops was provided by

the Office of Naval Research under Grants No. N00014.85-G-0219 and N00014.87-G-0231.

#### DEDICATION

In commemoration of the thirtieth anniversary of his first papers (published in the Physical Review) on maximum-entropy, the 1987 workshop and these proceedings are proudly dedicated to Edwin T. Jaynes. May his contributions continue for at least another thirty years.