

# Gravitation, Gauge Theories and the Early Universe

# Fundamental Theories of Physics

*An International Book Series on The Fundamental Theories of Physics: Their Clarification, Development and Application*

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# Gravitation, Gauge Theories and the Early Universe

*Edited by*

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# Preface

This book evolved out of some one hundred lectures given by twenty experts at a special instructional conference sponsored by the University Grants Commission, India. It is pedagogical in style and self-contained in several interrelated areas of physics which have become extremely important in present-day theoretical research. The articles begin with an introduction to general relativity and cosmology as well as particle physics and quantum field theory. This is followed by reviews of the standard gauge models of high-energy physics, renormalization group and grand unified theories. The concluding parts of the book comprise discussions in current research topics such as problems of the early universe, quantum cosmology and the new directions towards a unification of gravitation with other forces. In addition, special concise treatments of mathematical topics of direct relevance are also included.

The content of the book was carefully worked out for the mutual education of students and research workers in general relativity and particle physics. This ambitious programme consequently necessitated the involvement of a number of different authors. However, care has been taken to ensure that the material meshes into a unified, cogent and readable book. We hope that the book will serve to initiate and guide a student in these different areas of investigation starting from first principles and leading to the exciting current research problems of an interdisciplinary nature in the context of the origin and structure of the universe.

We are grateful to all the authors, who in spite of their busy professional commitments, gave us their precious time and enthusiastic support. It is a pleasure to thank Ms Moksha Halesh, Mr Ramachandra Rao and Mr Raju Varghese for patient and valuable help in preparing the manuscript for publication. And finally we acknowledge the cooperation and encouragement of our publishers Kluwer Academic Publishers, The Netherlands in bringing out this volume.