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Preface

The linear theory of oscillations traditionally operates with frequency representations based on the concepts of a transfer function and a frequency response. The universality of the critria of Nyquist and Mikhailov and the simplicity and obviousness of the application of frequency and amplitude - frequency characteristics in analysing forced linear oscillations greatly encouraged the development of practically important nonlinear theories based on various forms of the harmonic balance hypothesis [303]. Therefore mathematically rigorous frequency methods of investigating nonlinear systems, which appeared in the 60s, also began to influence many areas of nonlinear theory of oscillations.

First in this sphere of influence was a wide range of problems connected with multidimensional analogues of the famous van der Pol equation describing auto-oscillations of generators of various radiotechnical devices. Such analogues have as a rule a unique unstable stationary point in the phase space and are Levinson dissipative. One of the pioneering works in this field, which started the investigation of a three-dimensional analogue of the van der Pol equation, was K.O.Friedrichs's paper [123]. The author suggested a scheme for constructing a positively invariant set homeomorphic to a torus, by means of which the existence of non-trivial periodic solutions was established. That scheme was then developed and improved for different classes of multidimensional dynamical systems [131, 132, 297, 317, 334, 357, 358]. The method of Poincaré mapping [12, 13, 17] in piecewise linear systems was another intensively developed direction.

The application of the Yakubovich – Kalman frequency theorem [130, 154, 178, 267, 323, 372, 376, 382] to the analysis of quadratic forms generating a positively invariant torus led to new problems, the solution of which allowed the formulation of a number of frequency criteria for the existence of cycles in multidimensional analogues of the van der Pol equation [94, 180, 278, 280, 281, 338, 339, 341].

The ideas of E.D.Garber and V.A.Yakubovich [127, 381, 383] enable one to obtain frequency estimates of the period and "amplitude" of these oscillations. It should be noted that since frequency criteria for the existence of cycles are based on the Yakubovich – Kalman theorem, then for an estimate of the period the method of a priori integral estimates of V.M.Popov appears to be the most developed at present.

Other nonlinear effects qualitatively different from auto-oscillations in the van der Pol equation are observed in dynamical systems with angular coordinates. One can mention first of all circular motions and cycles of the second kind in the equation of a pendulum. Synchronous electrical machines and electronic systems of phase synchronization are described by the same equations [247, 330, 387]. The foundations of the nonlocal theory of two-dimensional systems with angular coordinates were laid in the works of F.Tricomi and his numerous followers [6, 39, 46, 61, 145, 328, 350]. However, a less rough idealization for synchronous machines and the complication of phase synchronization devices required the investigation of systems of higher dimension.

The synthesis of the Lyapunov direct method and the elements of bifurcation theory, as well as the construction of various comparison systems, turned out to be the most effective. The Lyapunov functions constructed in this case contain cycles and separatrices of the corresponding two-dimensional comparison systems. In this way it became possible to obtain frequency criteria for the existence of circular motions and various types of cycle, which extend the widely known theorems of Tricomi, Amerio and other authors to multidimensional systems [99, 130, 183, 184, 186, 195].

E.A.Barbashin and J.Ezeilo posed the problem of the existence of a cycle of a third-order differential equation with a cylindrical phase space describing various synchronization systems. From the control theory point of view the difficulty of investigating this equation is due to a certain degeneration of its transfer function. It is similar to critical cases in classical stability theory. The frequency criteria for the existence of cycles of the first and second kind are obtained in the works [89, 188, 192, 203], which in particular answer the questions put by Barbashin and Ezeilo.

The third current direction in the applied theory of oscillations is the investigation of cycles in dissipative systems with one locally asymptotically stable equilibrium. In 1949 M.A. Aizerman [4, 5] put forward the conjecture of stability in the large of multidimensional dynamical systems with one nonlinearity satisfying the generalized Routh-Hurwitz conditions. N.N. Krasovskii [169] was the first to refute this hypothesis, pointing out a two-dimensional system of this class which has solutions going to infinity. V.A.Pliss [296] proved the existence of cycles for a three-dimensional system and he was the first to obtain non-trivial upper estimates for a sector of absolute stability. Futher development of Pliss's method led to frequency criteria for the existence of cycles in multidimensional systems that satisfy the generalized Routh-Hurwitz conditions [179, 280].

Close to the results indicated come the frequency criteria for oscillation in systems with nonstationary and hysteretic nonlinearities [189, 226], extending the widely–known theorems of A.A. Andronov and N.N. Bautin [14], N.A. Zheleztsov (see [16]), A.A. Feldbaum [120], A.Yu. Levin [241], R.W. Brockett [74], E.S. Pyatnitskii [313] to the multidimensional case.

After E.N.Lorenz's [254] discovery of strange attractors a great many experimental and theoretical works appeared [18, 111, 135, 137, 268, 269, 275, 289, 315, 316, 335, 344, 345, 347], which made it clear that stochastic oscillations are widespread in finite-dimensional dynamical systems. In this case cycles do not have any significance in the system under consideration because of their instability and hence their physical unrealizability, even if they do exist in such attractors. Global characteristics such as various dimensions of attractors were advanced [275, 352]. Note that the dimension of a strange attractor in which chaotic oscillations occur is as important a quantitative characteristic of oscillations as its frequency in the case of ordinary periodic oscillation . The work of A.Douady and J.Oesterlé [112] was an important step in obtaining frequency estimates of the Hausdorff dimension [68, 69]. So there arose a close relationship between the procedure used in the articles mentioned and the works of G.E.Borg, F.Hartman, C.Olech, G.A.Leonov [73, 143, 208–210, 212, 284], in which the orbital stability of trajectories is investigated. It turned out that the problems of estimating the Hausdorff dimension and investigating orbital stability reduced to the local study of compressing properties of a shift operator along the trajectories of the systems under consideration.

By now it had become clear that, on the one hand, analytical methods developed for upper estimates of the Hausdorff dimension of attractors are a part of the modern theory of stability of motion. And on the other hand, the interpretation of the Hausdorff measure of compact sets mapped by a shift operator along trajectories as an analogue of the Lyapunov function allows one to obtain new results in the classical theory of stability. Such understanding especially stimulated the introduction of the notion of weakly contracting systems [148–150, 152] and the investigations of A. Douady and J. Oesterlé [112], R. Smith [340], R. Temam [351, 352], A.V. Babin and M.I. Vishik [22, 23]. Applying the frequency theorem of Yakubovich and Kalman [382], it is possible to give estimates of the Hausdorff dimension a frequency form [68, 69, 84, 189].

And finally the introduction of Lyapunov functions into estimates of the Hausdorff dimension of attractors by generalizing the estimates of Douady and Oesterlé [214, 215] made it possible to suggest a combination of classical theorems of the second Lyapunov method [101, 109, 130, 171, 259, 323] and theorems of Hartman, Olech and Smith [142, 143, 340].

Since "nonlinear frequency reasoning" is a rather difficult branch of the applied theory of differential equations, the authors have tried to present it as simply as possible for a majority of readers. With this aim there are two introductory chapters. In the first chapter two-dimensional oscillation systems and their multidimensional analogues are considered and discussed. In the second chapter a short summary of the main results on frequency criteria for absolute stability and quadratic matrix inequalities is given.

The third chapter is devoted to the investigation of multidimensional analogues of the van der Pol equation. The fourth chapter gives frequency estimates of the period and amplitude. In the fifth and the sixth chapters a frequency approach to the study of dynamical systems with cylindrical phase space is presented.

The seventh chapter considers problems connected with the conjecture of Aizerman.

In the eighth chapter attention is concentrated on estimates of the Hausdorff dimension of attractors and methodologically close questions of Poincaré and Zhukovsky stability of trajectories.

The beginning of the third and the fifth chapters may seem unnecessarily long for the specialist. But we intend this book for the reader who has just begin to study the frequency analysis of nonlinear systems.

It should be noted that the authors have focused only on oscillations in autonomous systems. This is due to the fact that mathematically rigorous methods of frequency analysis of forced nonlinear oscillations do not exceed for the time being the bounds of the classical theory of absolute stability, and are well discussed in the literature [244, 267, 384].

The two-digit system of numbering formulae, theorems, definitions, examples and figures is used in the book. When they are mentioned in other chapters a figure denoting the number of the respective chapter is added.

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