

QUALITY IMPROVEMENT WITH DESIGN OF EXPERIMENTS

TOPICS IN SAFETY, RISK, RELIABILITY AND QUALITY

VOLUME 7

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Aims and Scope. Fundamental questions which are being asked these days of all products, processes and services with ever increasing frequency are:

What is the risk?

How safe is it?

How reliable is it?

How good is the quality?

How much does it cost?

This is particularly true as the government, industry, public, customers and society become increasingly informed and articulate.

In practice none of the three topics can be considered in isolation as they all interact and interrelate in very complex and subtle ways and require a range of disciplines for their description and application; they encompass the social, engineering and physical sciences and quantitative disciplines including mathematics, probability theory and statistics.

The major objective of the series is to provide a series of authoritative texts suitable for academic taught courses, reference purposes, post graduate and other research and practitioners generally working or strongly associated with areas such as:

Safety Assessment and Management

Emergency Planning

Risk Management

Reliability Analysis and Assessment

Quality Assurance and Management

Special emphasis is placed on texts with regard to readability, relevance, clarity, applicability, rigour and generally sound quantitative content.

The titles published in this series are listed at the end of this volume.

Quality Improvement with Design of Experiments

A Response Surface Approach

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To

Irina, Maria, Ivan

and

Ilia, Boyan, Kamen

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PREFACE

This book is devoted to the problem of quality improvement of products and processes through robust engineering design. Taguchi was a pioneer in this field and his methodology became popular among the engineers and statisticians. Many successful applications were reported. While Taguchi's engineering ideas were widely recognized, many statisticians expressed criticism with respect to his statistical procedures. This inspired development of an alternative to Taguchi's approach based on response surface methodology.

High quality of a product can be achieved when the mean values of its performance characteristics are close to given targets, while the variations are as small as possible. The fundamental idea behind this book is to use models to this end. Experiments have to be carried out in order to collect information for model building. Response surface methodology provides a variety of economical designs that can be used for quality improvement. Two models can be obtained on the basis of observations – one for mean value and another for variance of product's performance characteristic. With these models optimization procedures can be used to find product parameters that minimize performance characteristic's variance while keeping the mean value on a target.

The book integrates design of experiments, model building and optimization techniques for robust product or process design. The first chapter is introductory and gives a general idea for model-based robust engineering design.

In order to make the book self-contained the basic ideas of the response surface methodology are given in chapters 2 and 3. Chapter 2 presents statistical methods for data analysis. Analysis of variance and some important combinatorial experimental designs are considered. They are used in Chapter 4, where Taguchi's methodology for quality improvement is presented. In Chapter 2 are also given basic principles of the regression analysis. This is the main model-building tool used in the response surface methodology.

In Chapter 3 we consider some important response surface designs. Methods for interpretation of the models obtained through regression experiments are also discussed. This is a basis for understanding the material given in Chapters 5 to 10.

Chapter 4 presents the main ideas and tools for quality improvement through design of experiments proposed by Taguchi. On this basis a model-based approach to this problem was developed, which is an attempt to combine the engineering ideas of Taguchi with response surface methods for model building and optimization.

As it was noted, model-based robust engineering design requires two models: one for the mean value and another for the variance. They take into account two sources of variation: errors in product parameters and external noise factors. In the next chapters is shown that the models reflecting the errors in product parameters and the external noise factors have different properties. In Chapter 5 models of the mean value and variance

induced by the errors in product parameters are considered. The mechanism of error transmission from the product parameters to the response is studied. An analysis of the accuracy of these models is also presented.

Chapter 6 proposes optimization procedures for robust design based on models of products or processes with errors in parameters. Optimization criteria are discussed. Analytical and numerical optimization methods are considered, model-based decision making and tolerance design procedures are proposed.

Chapter 7 deals with methods for design of experiments, model building and optimization for products and processes, which are subject to both errors in parameters and external noise factors.

Models of mean value and variance of the performance characteristics induced on the basis of mechanistic models of products or processes are considered in Chapter 8. If there are errors only in product parameters these models can be obtained without experiments or simulations of noises. Methods for incorporating experimental data in the mean and variance models are given in this chapter.

Models for quality improvement of products and/or processes with qualitative and quantitative factors are proposed in Chapter 9. Optimization procedures are given as well.

Methods for building mean and variance models based on repeated observations in the design points and their modifications for unreplicated observations are considered in chapter 10. Graphical tools for studying location and dispersion effects are given. Parameter estimation in cases with errors in factor levels is briefly presented. At the end of this chapter attention is paid to a model-based approach to robust design of signal-dependent systems.

The book has been written for engineers and statisticians working in the field of quality improvement and for senior and graduate students in engineering. Previous versions of the text have been used as a textbook for students and in industrial short courses.

It is assumed that the readers possess knowledge of basic statistical methods and matrix algebra. Through all chapters the main ideas are presented avoiding complicated mathematical proofs. They are explained through many real and constructed examples. For readers who want to go in details the proofs are given in appendices to most of the chapters. Additional reading in the end of each chapter and an extensive bibliography in the end of the book are provided.

The idea of writing the book came to us from a short course organized by Dr. E. Walter for industrial engineers in SUPELEC, Paris where we presented the main ideas of model-based quality improvement. We would like to thank Prof. Henry Wynn, Dr. Eric Walter and Dr. Luc Pronzato for their helpful discussions during the research activities that lead to the development of this book. The authors very much appreciate the editorial assistance of Mrs. Milena Todorova, who greatly improved the readability of the book.

We are most grateful to our families for their continuous encouragement of our research and teaching activities that made possible the appearance of this book.