The Dialectic Relation Between Physics and Mathematics in the XIXth Century

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The Dialectic Relation Between Physics and Mathematics in the XIXth Century

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Foreword

Mathematical Physics and Theoretical Physics as Dialectics Between Mathematics and Physics

From the beginning of the nineteenth century, the scientists themselves are explicitly concerned by the problems arising from the relations between Mathematics and Physics. Their different answers constituted the different kinds of what they called “Mathematical Physics” and “Theoretical Physics”. Indeed, many of their reflections focused on certain antinomies or dualities like oppositions between continuum and atoms, dualities between facts and laws, empiricism and rationalism, reasoning and theoretical foundation, and also turned into confrontations, like between atomism and energetism, mechanism and functionalism. In a more precise sense, the constructions of Mathematical Physics and Theoretical Physics were the answers that they gave to dissolve (analysis) or to thematize (synthesis) the antinomies and the oppositions, in any case to pass over them by the invention of mathematical or physical concepts, mathematically or structurally linked. It is under this last condition that the dualities became dialectics.

The historical approach permits to analyze the conditions of the births of the answers, concepts and theories, and to examine their explicit confrontations. Moreover, this approach shows the manners that are used by scientists to adapt the concurrent positions and so to create new conceptions. The different answers are constructed by scientists, heirs of scientific or philosophical schools and gathered in universities or nations. This book proposes to take a European scale to analyze many examples of dualities, confrontations and dialectics and to try to have a more global view. Indeed, there was a plurality of new conceptions, but there was also a circulation between them. This circulation did not only depend of the possibilities of communication at this period, but also of the same willingness showed by the scientists to obtain a unity of the different physical sciences.

Let us examine some of these aspects in the light of the different chapters of this book. From the beginning of the nineteenth century, many difficulties occurred in the mathematical treatment of the phenomena of Nature, despite the success of the methods of the previous century. One of them was the lack of simplicity of the mathematics involved in physical sciences, particularly because it seemed not possible to realize them in an immediate perception.

For René-Just Haüy, the research of simple results corresponded to his conviction on the simplicity of Nature. It served explicitly to deduce the "true ratio" between measurements of angle, in his work on crystallography of 1801 (Chap. 1). Samuel Christian Weiss, an heir of the German philosophy, criticized Haüy's conception because it presents two defaults, the atomism and the empiricism, on which it depends. Weiss overcame the opposition between molecule and continuum and the conflict between empiricism and rationalism, by introducing a simple mathematical concept, which is the axis of symmetry (1814).

While for James Clerk Maxwell, the simplicity was not to seek in the things of Nature but in the process of the reasoning. Moreover, this simplification may take the form of "a purely mathematical formula" or of "a physical hypothesis" (Chap. 2). Here, a duality opposes the process of reasoning to the theory of knowledge. His criticism of Ampère's works on electro-magnetism shows the kind of rupture he introduced. Maxwell considered that the reasoning of Ampère on infinitesimal elements is not sufficient to show the electro-magnetic actions. For him, the connection between two sciences has to depend on a "physical analogy" (1856), that means a similarity established by the reasoning between the laws of one science and those of another. This programme will have an achievement with the concept of electromagnetic field (1865).

The confrontation between English and continental conceptions gives other examples of the resolution of dualities. The researches on the theory of elasticity in Italy are interesting from this point of view, because they concern the opposition between atomism and energetism (Chap. 3). In his first work on elasticity (1866), Enrico Betti assumed bodies as formed by molecules, and he introduced a potential function with consideration of forces. But, the next year, on the basis of William Thomson's works, he gave an energetic meaning to the potential. In his book, entitled *Theory of elasticity*, the exposition begins with the principles of potential energy and virtual work.

The meeting between Joseph Liouville and William Thomson in Paris in 1845 had important consequences on the kind of mathematics elaborated by the former in connexion with Physics, and so on his conception about relations between Physics and Mathematics (Chap. 4). Indeed, Liouville considered in 1832 that the solution of most physico-mathematical problems only consists in solving differential equations. But, in Paris, Thomson showed to his French colleague a transformation that maps an electrostatic problem into another problem easier to prove. One consequence will be the geometrical theorem of Liouville on conformal mappings of space given in 1850.

The historical approach indicates many fields where the willingness of unity expressed itself, from the active reasoning of the scientist to his abstract speculations. In the different physics, is it possible to always use the same reasoning for the English Maxwell, the same mathematics for the French scientist Gabriel Lamé or the same principles for the German scientist Carl Neumann. For this purpose, the context of the writing of treatises is important to analyze the implemention of the unity of physical sciences, like we can see in two chapters devoted to French and German universities.

With Gabriel Lamé, the work on a unified Mathematical Physics is linked with his lessons given to students in the university of La Sorbonne in Paris in the year 1850. He proposed a mathematical unity with the invention of curvilinear coordinates, and for him the truly universal principle of Nature is aether. His pupil Emile Mathieu conceived the project of a complete treatise when he gave lessons in the university of La Sorbonne in Paris (Chap. 5). For him, the unity could not be found in the aether, but in the unique procedure, which operates in each of the physical sciences. He intended to work with a minimum of physical principles, and a strong mathematical theory, which is developed in the first volume of the treatise in 1873. The Mathieu's treatise, edited from this date until 1890, took into account the different works done before and research to adapt them to his own conception. For instance, he quoted Maxwell as well as Wilhelm Weber and Carl Neumann.

Neumann expressed his point of view on Physical Theory in his inaugural lectures given in Tübingen and then in the university of Leipzig in 1869. For him, the task of the physicist is to explain all phenomena that occur in Nature by a few basic principles as possible and a few inexplicable facts as possible (Chap. 6). The proper aim of mathematics consists in finding out the principles from which the laws of empirical facts can be derived in a mathematically correct way, and so to find out principles "equivalent" to the empirical facts.

One result of the conceptions of Neumann and Mathieu is that the theories appeared to be temporary, because they depend of the mathematics involved by the scientist and by the facts he could or he decided to gather. In 1870, Neumann conceived that a physical theory is always incomplete and changeable. Some years later, Mathieu presented the theories in their historical process and gave historical developments, which lead to them, in his treatises. The idea that the theories depend of the historical process is present in the epistemologies of the beginning of the twentieth century, like those of Pierre Duhem in 1906 or Ernst Cassirer in 1910. For Duhem, there is a continuum of progress in history, while for Cassirer there exists a fundamental rupture in the historical passage from a science thought as a description of perceptions to a science constructed as a set of mathematical relations.

The Chap. 7 of this book propose two epistemological reflections on theoretical Physics. The first one considers that "physical world" force, pressure and entropy always existed and that their properties are independent from the development of our physical knowledge (Chap. 7). But "the world of physics" has undergone a number of fundamental changes and the most important of them concerns the language of physics. On this basis a reconstruction of the historical development of the language of physical theories is proposed. It is the case for the field theory from 1831 to the end of the century.

The Chap. 8 emphasizes that the pluralism in the relationship between Physics and Mathematics in the nineteenth century was disregarded (Chap. 8). For instance, it is noted that the date of "birth" of each new theory of the century was often contested. After a theory was considered as completed, some foundational parts were added some decades later, like for the thermodynamics in 1850, which was seriously modified in 1893 by Planck and in 1896 by Mach. This could be interpreted by the fact that the "true differences" among the various relations between mathematics and physics were ignored. This can also explain the crisis in the first years of the twentieth century. The reason would be that the community of physicists was unaware of the profound changes occurring at the time. This interesting historical and epistemological problem is surely one of the questions, which can serve as a vital lead to read this book.

One possible conclusion of this book can be that the important differences between conceptions have not to mask to us a major agreement between scientists on the necessity to work on the relations between mathematics and physics themselves. For many French scientists, the Mathematical Physics consisted in associating a differential equation to a phenomena and to solve it. So, the physical sense of a differential equation supposed an interpretation of the symbols in terms of physical notions (potential, time, etc.), and a physical science consisted in linking symbols and notions. While the physical analogy of Maxwell supposed the establishment of a relation between notions of two physical sciences, it created a link between physical notions. In the first case, the mathematical connections between physical notions were those given by the calculus on magnitudes (continuum), while in the second case, the mathematical connections could be purely relational.

In any case, the correspondence between laws and facts became a dialectic expressed in the terms in which the task of mathematics is given. Now the purpose was not to link physical notions to things but to link physical concepts together by mathematical relations. The function of the concept is to gather and to order the empirical material. There were two mathematically ordered series in correspondence: series of facts and series of physical concepts. In 1905, Henri Poincaré explained that if the experience made known a relation between bodies, A and B, which was complicated, then we could introduce three intermediary relations: a relation between A and a figure A', a relation between B and a figure B' and finally a mathematical relation between A' and B', which had the property to be simple. So, mathematical relations were the instruments to construct simplicity. He explained that science was a classification and precisely a system of relations. For him, the objectivity had to be found in these relations and not in the isolated things.

This book takes into account most of the contributions of the Symposium organized by Evelyne Barbin and Raffaele Pisano in the 4th International Conference of the European Society for the History of Science (ESHS), which was held in Barcelona from 18th to 25th of November 2010. It was specially prepared with the purpose to make known some recent developments in the history of science and to share the interesting exchanges of the symposium.

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