

# Design, Modeling and Experiments of 3-DOF Electromagnetic Spherical Actuators

# MECHANISMS AND MACHINE SCIENCE

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*The book is dedicated to authors' families.*

# Preface

学而不思则罔，思而不学则殆。  
— 孔子（公元前551-479）

*Learning without thought is labor lost; thought without learning is perilous.*  
— Confucius (551-479 BC)

Motors and actuators producing one degree-of-freedom (DOF) rotary or linear motion have been the main driving elements of modern machinery for a long time. With the advancement of technology, more and more new machines possessing complex structures and sophisticate motions to achieve tasks that have never been automated before have been developed, for example, humanoid robots, surgical robots, electric vehicles, just to name a few. In order to have a compact design of the machinery, actuators that can produce multiple degrees of freedom motion in one integrated package without the intermediate transmission mechanisms become crucial. However, there are quite a few technical challenges to realize a multiple DOF actuator at present. First, investigation of the actuating principle for multiple DOF motions with load bearing capability like or greater than the human is necessary. There are very few types of actuators that can achieve the load density comparable to that of the human. The design of the load bearing structure for multiple DOF motion is another important factor. One DOF rotary or linear motors can run smoothly and precisely with readily available precision rotary and linear bearings. It is not possible to find such component for multi-DOF actuators at present. If precise motion control is required for such multi-DOF actuators, sensors that can detect the multiple DOF rotor motion are necessity. Hence, multi-DOF sensing technology becomes part of the work for the realization of the actuator.

Spherical motion is a compact multi-DOF rigid body motion along a spherical surface with permanent center of rotation. By examining the machinery and systems surrounded us, spherical motion could possibly be the second most important motion type next to the compact rotary motion. Biological systems examples with spherical motions can be found in the eyeball movements, human wrist, shoulder,

and hip joint motions. Industrial examples can be found in the surveillance devices, automation tools, automobiles, and home appliances. Existing spherical motion generation devices are frequently designed by combining two to three rotary actuators with serially or parallel connected mechanisms. Such devices are inevitably bulky and difficult to be deployed as a critical component in complex machinery. Therefore, the need to develop a multiple DOF actuator capable of generating spherical motion arises.

The theme of this monograph is on the development of a compact electromagnetic spherical actuator that can produce two or three degrees of spherical motion. Through the study of this spherical actuator, the challenges of designing a multi-DOF actuator mentioned above are to be addressed. Spherical actuators based on a number of actuating principles such as electromagnetic force, piezoelectricity and mechanical means, have been explored by researchers. Electromagnetic actuation principle is chosen because of the load density, abundance of the material, and its compatibility with existing single DOF motors widely available at present. The underlying actuator design is a ball-joint structure with a rotor and a stator. The rotor is capable of moving in different orientations with finite angles or full 360 degrees about a fixed point within the actuator. It is our hope that the spherical actuator designed in this manner can be accepted and adopted by the industry based on its novelty as well as its compatibility and resemblance with existing motors in terms of the actuating principle and control.

This monograph presents a generic methodology on modeling, design and experimental investigation on 3-DOF permanent magnet (PM) spherical actuators. A parametric design approach is adopted from the coil and pole layout, spherical magnetic field strength to the motor torque-current function. This approach offers researchers and engineers to design the spherical actuator, such as the pole configurations and structure parameters, based on specific requirements from the applications. Furthermore, the actuator design can be fine-tuned and optimized, which is often required in complex system design. The topics in this book cover the introduction of different types of spherical actuators, formulation of three-dimensional (3D) spherical magnetic field, analytical motor torque modeling, prototype development, experimental investigation on magnetic field and motor torque, and 3D orientation sensing techniques.

While this book is primarily intended for researchers and developers working on spherical actuators, sensors and instruments, we also hope that it will interest researchers and students working in the area of electric machines since the methods proposed could also be useful for the understanding and analysis of others electromagnetic actuators; people in the area of magnetic field analysis since this book is also an illustrative example of the modeling and analysis of complex magnetic fields; people in the area of sensors since this book provides a solid starting point for the basic concept and working principle of various orientation measurement methods. It is also our hope that this book could provide a steppingstone for greater technology advancement in multi-DOF actuator design as future applications of these actuators are abundant.

This book is a complete exposition of the research work accomplished by the authors on the electromagnetic spherical actuators over the years through research collaboration among the Nanyang Technological University (Singapore), Singapore Institute of Manufacturing Technology, and Georgia Institute of Technology (USA). The authors sincerely appreciate the institutional support received from the three organizations. The major portions of the book were based on the PhD dissertations of two authors, Liang Yan and Chee Kian Lim while studying at Nanyang Technological University. Part of the work on orientation sensing for spherical actuators received graduate student manpower support from China Scholarship Council. This book would not have been possible without the help and the support from many people. In particular, we would like to thank Dr. Wei Lin for the financial support, Mr. Jialin Su for valuable technical comments, Prof. Weihai Chen, Prof. Yunhua Li, Dr. Zhongwei Guo and Dr. Yan Lang for the design and construction of the experimental apparatus and the actuator control system, and Thomas Moong for data collection, in carrying out the research project and subsequently completion of this book. Deep appreciations from the authors also go to several former members of the Robotics Research Center of Nanyang Technological University, especially Yan Jin and Xueyan Tang for a number of figures in this book. The authors are grateful to their families whose patience and comprehension have permitted them to spend time and effort on elaborating and completing this book. Finally, the authors would also like to acknowledge the support from the Agency for Science, Technology and Research (A\*STAR) of Singapore, the National Nature Science Foundation of China, and the Fundamental Research Funds for the Central Universities, China.

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# Acronyms

1/2/3D	One/two/three-dimensional
AC	Alternating current
AWG	American wire gauge
BC	Boundary condition
CCD	Charge-coupled device
DAQ	Data acquisition
DC	Direct current
DOF	Degree-of-freedom
DLIM	Double-sided linear induction motor
FE	Finite element
FEA	Finite element analysis
SLIM	Single-sided linear induction motor
MST	Maxwell stress tensor
PC	Personal computer
PM	Permanent magnet
PZT	Piezoelectric
SPM	Spherical pointing motor
SWM	Spherical wheel motor
SRM	Switched reluctance motor
VRSM	Variable-reluctance spherical motor