Understanding Pendulums

HISTORY OF MECHANISM AND MACHINE SCIENCE

Volume 12

Series Editor

MARCO GIOVANNI CECCARELLI

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This book series aims to establish a well defined forum for Monographs and Proceedings on the History of Mechanism and Machine Science (MMS). The series publishes works that give an overviewof the historical developments, from the earliest times up to and including the recent past, of MMS in all its technical aspects.

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L.P. Pook

Understanding Pendulums

A Brief Introduction

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Preface

My interest in pendulums goes back to the late 1940s when I saw the Foucault pendulum at the Science Museum in London. Soon after we were married in 1960 my wife and I bought a longcase clock, which we still have. Keeping this, and other pendulum controlled clocks, running has taught me a lot about clock pendulums and the escapements used to control them, and aroused my interests in theoretical aspects of pendulums. For some years I have been collecting a broad range of information on pendulums, and this book is the result. Writing the book would not have been possible without consulting reference material held by the British Library in London, whose staff have been invariably helpful.

There are three related difficulties in writing about pendulums. The first is that it is an enormous subject so that a brief introductory book has to be selective in the material to be included. The second is that detailed explanations of pendulum behaviour often require advanced mathematics that can be difficult to follow, and may not be available for some situations. The third is that, although it is possible to define an ideal simple pendulum, physical pendulums inevitably deviate from an ideal, and it is difficult to assess the significance of these deviations. Despite their apparent simplicity, the behaviour of pendulums can be remarkably complicated. Historically, pendulums for specific purposes have been developed using a combination of simplified theory and trial and error.

Books on scientific subjects can be written at three different levels, firstly at a popular science level, with little or no significant mathematical content. Secondly, at an intermediate level with mathematical content at a recreation mathematics level, and used to illuminate the subject. Finally, written at an advanced level for specialists, with the mathematical content needed to understand the material at a serious mathematics level. There do not appear to be any introductory books on pendulums, written at an intermediate level, and covering a wide range of topics. This book aims to fill the gap. It is written for readers with some background in elementary geometry, algebra, trigonometry and calculus. Historical information, where available and useful for the understanding of various types of pendulum and their applications, is included. The wide range of topics covered means that the material is, to some extent, arranged in an arbitrary order. Extensive cross references are included so that individual chapters do not have to be read in order. Definitions are included in the index so that they can be easily located. Examples are given as case studies, sometimes with qualitative rather than quantitative explanations.

As described in Chap. 1, perhaps the best known use of pendulums is as the basis of clocks in which a pendulum controls the rate at which the clock runs. Interest in theoretical and practical aspects of pendulums, as applied to clocks, goes back more than four centuries. The concept of simple pendulums, which are idealised versions of real pendulums is introduced in Chap. 2, together with their analysis, and some variations on simple pendulums are described in Chap. 3. The application of pendulums to clocks is described in Chap. 4, with detailed discussion of the effect of inevitable differences between real pendulums and simple pendulums. In a clock, the objective is to ensure that the pendulum controls the timekeeping. However, pendulums are sometimes driven and how this affects their behaviour is described in Chap. 5. Pendulums are sometimes used for occult purposes. It is possible to explain some apparently occult results by using modern pendulum theory. For example, the chapter includes an explanation of why a ring suspended inside a wine glass, by a thread from a finger, eventually strikes the glass. Pendulums have a wide range of uses in scientific instruments, engineering, and entertainment. Some examples are given as case studies in Chaps. 6, 7 and 8.

January 2011 Les Pook

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Notation

a	acceleration
a, b	Lissajous figure parameters, major and minor semi axes of ellipse
A	amplitude
A, B	arbitrary constants
A_1, A_2	arbitrary constants, amplitudes of successive peaks
B	force due to buoyancy
c	half diagonal of square
f	natural frequency, resonant frequency
f_p	frequency of periodic force
f_R	relative frequency
F	restoring force per unit distance
F_p	periodic force
$F(\beta)$	circular error at amplitude β
g	acceleration due to gravity, acceleration due to gravity at sea level
g'	acceleration due to gravity above sea level
g_e	effective value of g
$G(f)$	spectral density function
h	distance from frictionless pivot to centre of mass, a constant
H	height of point mass above its rest position
H_f	maximum value of H
I	moment of inertia, irregularity factor
j	distance from frictionless pivot to centre of oscillation
k	damping constant
K	kinetic energy
l	length of a simple pendulum, subscripts 1, 2 denote components of a double pendulum, effective length of a compound pendulum
l_V	vertical distance from pivot, virtual rod length
L	length, latitude
m	mass, subscripts 1, 2 denote components of a double pendulum
m_0, m_2, m_4	moments of spectral density function

m_V	velocity of point mass, m
M	mass
n	number of cycles, number of to and fro swings
N	number of balls in a Newton's cradle
p	momentum
$p(S)$	probability density
P	pendulum period $= T/2$, subscripts denote observed periods
$P(S)$	exceedance
Q	pendulum quality, quality factor
r	radius, radius of sphere, perpendicular distance from a pivot.
r, θ	polar coordinates
r, θ, ψ	spherical polar coordinates
r_s	radius of small circle
R	resisting force per unit velocity
RMS	root mean square
R_V	velocity of point R
$R(\tau)$	autocorrelation function
S	random process, positive peak, displacement from rest position, subscripts 1, 2 denote components of a double pendulum
SDF	spectral density function
S_m	Mean value of S
t	time
T	time of swing of a pendulum, period of simple harmonic motion or damped harmonic motion, time of one rotation about an axis, wave passing period, total time
T_E	effective time of rotation
V	velocity, strike velocity
W	potential energy
W_a	absorbed energy
x	distance from a fixed point, displacement from rest position
x, y	Cartesian coordinates, subscripts 1, 2 denote displacements of a double pendulum
X	pivot displacement
α	ratio between periods
$1/a$	relative frequency
α, ε	arbitrary constants
β	angle of swing of pendulum from rest position, maximum value of θ, angle of fall
γ	angular interval, angle of rise
δ	logarithmic decrement
ε	phase, spectral bandwidth
$\varepsilon_1, \varepsilon_2$	arbitrary constants
κ	radius of gyration
κ_m	radius of gyration about centre of mass
ϕ	phase angle, root mean square value of S

ϕ^2	mean square value of S
σ	standard deviation
σ^2	variance
τ	time constant, increment of time
θ	pendulum angle, subscripts 1, 2 denote components of a double pendulum, angle of rotation
ω	angular velocity (angles in radians)
ω_d	damped angular velocity
ω_p	angular velocity of periodic force

About the Author

Leslie Philip (Les) Pook was born in Middlesex, England in 1935. He obtained a BSc in metallurgy from the University of London in 1956. He started his career at Hawker Siddeley Aviation Ltd, Coventry in 1956. In 1963 he moved to the National Engineering Laboratory, East Kilbride, Glasgow. In 1969, while at the National Engineering Laboratory, he obtained a Ph.D. in mechanical engineering from the University of Strathclyde. Dr. Pook moved to University College London in 1990. He retired formally in 1998 but remained professionally active in the fields of metal fatigue and fracture mechanics, and was a visiting professor at University College London until 2009. He now has more tine to pursue long standing interests in recreational mathematics, including flexagons, and in horology, especially synchronous electric clocks. He is a Fellow of the Institution of Mechanical Engineers, a Fellow of the Institute of Materials, Minerals and Mining, and a Fellow of the European Structural Integrity Society. Les married his wife Ann in 1960. They have a daughter, Stephanie, and a son, Adrian.