

# History of Rotating Machinery Dynamics

# HISTORY OF MECHANISM AND MACHINE SCIENCE

Volume 20

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MARCO CECCARELLI

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J.S. Rao

# History of Rotating Machinery Dynamics

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Dedicated to the memory of my parents

Jammi Chikka Rao  
Jammi Ramanamma

# Contents

<b>Foreword</b> .....	xi
<b>Preface</b> .....	xv
<b>Acknowledgements</b> .....	xix
<b>1 Beginnings of the Wheel</b> .....	1
<b>References</b> .....	4
<b>2 Science before the Medieval Period</b> .....	5
<b>References</b> .....	7
<b>3 Water Wheels</b> .....	9
<b>References</b> .....	12
<b>4 Wind Mills</b> .....	13
<b>References</b> .....	14
<b>5 Renaissance and Scientific Revolution</b> .....	15
<b>References</b> .....	20
<b>6 Renaissance Engineers</b> .....	23
<b>References</b> .....	30
<b>7 Industrial Revolution</b> .....	31
<b>References</b> .....	34

<b>8</b>	<b>Turbomachines</b> .....	35
	<b>References</b> .....	42
<b>9</b>	<b>Fundamentals of Elasticity</b> .....	45
	<b>References</b> .....	46
<b>10</b>	<b>Energy Methods</b> .....	49
	10.1 Euler–Lagrange Equations .....	50
	10.2 Lagrange Method .....	55
	10.3 Rayleigh’s Energy Approach .....	56
	10.4 Ritz Method .....	58
	10.5 Lagrange Method for Vibration Problems .....	61
	10.6 Galerkin Method .....	63
	10.7 Hamilton’s Principle .....	66
	10.8 Complementary Virtual Work .....	78
	10.9 Hellinger–Reissner Variational Principle .....	81
	10.10Hu–Washizu Principle .....	87
	10.11Different Theories of Torsion of Rods .....	90
	10.11.1Coulomb (1784) Elementary Theory , see Timoshenko and Goodier [42] for Circular Rods .....	90
	10.11.2St. Venant (1853) Theory, see Todhunter [43] and Timoshenko and Goodier [42] for Circular Rods .....	90
	10.11.3Love’s (1944) Theory .....	91
	10.11.4Timoshenko (1945) – Gere’s (1954) Theory .....	91
	10.11.5Reissner (1952) and Lo–Goulard’s (1955) Theory .....	92
	10.11.6Barr’s (1962) Theory .....	93
	10.11.7Refined Theory by Rao (1974) .....	93
	<b>References</b> .....	97
<b>11</b>	<b>20th Century Graphical and Numerical Methods</b> .....	99
	11.1 Stodola–Viannello (Rayleigh’s Maximum Energy) Method in Graphical Form .....	99
	11.2 Stodola–Viannello Iterative Method in Tabular Form .....	101
	11.3 Dunkerley’s Method .....	104
	11.4 Proof of the Dunkerley Formula by Blaess [1] .....	104
	11.5 Hahn’s Proof Using Matrix Algebra [3] .....	105
	11.6 Holzer Method for Torsional Vibration .....	107
	11.7 The Myklestad Method [7, 8] .....	108
	11.8 Prohl’s Method [9] .....	112
	<b>References</b> .....	114

<b>12 Matrix Methods</b> .....	115
12.1 Torsional Vibration Systems .....	118
12.2 Far-Coupled Systems .....	121
12.3 Gräffe's Method of Successive Approximations .....	122
12.4 Matrix Iteration Method .....	124
12.5 Method of Priebs [10] .....	127
12.6 The Holzer Method (Close Coupled Systems) in Transfer Matrix Form .....	131
12.7 Myklestad–Thomson (1949, 1953) – Prohl Methods in Transfer Matrix Form for Far-Coupled Systems .....	133
12.8 A Brief Note on Computers and Evolution .....	136
<b>References</b> .....	138
<b>13 Finite Element Methods</b> .....	141
13.1 Beam Finite Element .....	143
13.2 Tocher Triangular Plate Element (1962) .....	147
13.3 Shell Element .....	154
13.4 Interface Damping through Finite Element Analysis .....	163
13.5 Illustration of Turbomachine Blade Analysis using Commercial Codes .....	176
<b>References</b> .....	183
<b>14 Rotor Dynamics Methods</b> .....	185
14.1 De Laval Model .....	186
14.2 Jeffcott Rotor Analysis .....	188
14.3 Fluid Film Bearings .....	190
14.4 Oil Film Instabilities .....	198
14.5 Quality Factor .....	204
14.6 Gyroscopic Effects .....	207
14.7 Internal Friction, Hysteresis .....	212
14.8 Shafts with Gravity and Variable Elasticity .....	218
14.9 Misalignment .....	226
14.10 Bowed Rotors .....	230
14.11 Variable Inertia .....	233
14.12 Seals and Instabilities .....	236
14.13 Steam Whirl .....	240
14.14 Cracked Shafts .....	242
<b>References</b> .....	248
<b>15 Transfer Matrix Methods</b> .....	253
15.1 Torsional Vibration due to Short Circuit of Generators .....	253
15.2 Transfer Matrix Method for Lateral Vibrations of Rotors .....	258
15.3 Twin Spool Rotor Analysis .....	262



<b>References</b> .....	267
<b>16 Finite Element Methods for Rotor Dynamics</b> .....	269
16.1 Nelson's Beam Element .....	269
16.2 Geared Rotors and Chaos .....	277
16.3 Solid Rotors .....	284
16.4 Two Spool Aircraft Engine [21] .....	289
16.5 Cryogenic Pump Rotor Dynamic Analysis .....	289
<b>References</b> .....	295
<b>17 Bladed Disks</b> .....	299
17.1 Armstrong's Analysis for Tuned Systems .....	300
17.2 Ewins' Analysis .....	302
17.3 Mistuning Arrangement .....	306
17.4 Damping .....	310
17.5 Micro-Slip Damping (Fretting Fatigue) .....	314
<b>References</b> .....	323
<b>18 Lifting</b> .....	327
18.1 High Cycle Fatigue (HCF) Life Estimation .....	331
18.2 Low Cycle Fatigue (Strain Based Life Estimation) .....	333
18.3 Linear Elastic Fracture Mechanics .....	333
<b>References</b> .....	338
<b>19 Optimization</b> .....	341
19.1 Shape Optimization .....	343
19.2 Weight Optimization .....	346
<b>References</b> .....	351
<b>20 Concluding Remarks</b> .....	353
<b>Index</b> .....	355

# Foreword

Human history may be built on the development of technology, but the history of rotordynamics to date has been a blank page of that history. Dr. Rao's book will do much to fill this empty space. Engineers are not historians, but it seems that wherever the history of a specialty is written, it is a specialist – Dr. Rao in this case – who must rise and address this need. Clearly, the history of any specialized area presents a very difficult task for anyone other than the expert himself.

Fortunately, Dr. Rao has chosen to address this task, a task it must be said that he is well-equipped to undertake.

Recognizing the blank page which stood before him, Rao has done well to first put his efforts into a broad historical perspective before attempting the specialized areas that make up the complex science of rotor-dynamics. For this is a subject which extends from shaft dynamics, through the static and dynamic properties of various types of bearings and seals, and often involves both blade and disk dynamics. A well-established basis of knowledge in many technologies is therefore needed before one could think of considering the history of all these subjects in any depth. Small wonder then that this task has lain unaddressed for years, waiting for a specialist like Dr. Rao, who has spent a lifetime of scholarship and preparation before attempting a history of a subject as broad as this.

Rao begins with chapters on the wheel, the water wheel and on the windmill. Recognizing the contributions that Aristotle and later Archimedes made to the science which underlies the technology of rotating machinery, he discusses the contributions of the great engineers of ancient times, and he mentions the fallacies that occasionally grew from such work, even from the teachings of the great Aristotle and Ptolemy. In this, Rao subtly poses an interesting question: is it right to expect that even from birth that science shall be perfect? Evidently not: rather, we come to realize through our own work the value of such a book as this, that the creation of science and technology, is an on-going task, and one for which we must all assume some responsibility through our own publications. From the Ancients Rao next moves through the aridity of the Dark Ages in the West, through the Renaissance to the great re-generation of science that began with the work of certain

inspired scientists, and on to the emergence of the new technology of the Industrial Revolution, and to its flowering in the service of man.

The pace quickens as Rao finds himself in familiar territory, and he begins to address the foundations of the modern science of rotating machinery dynamics. The many methods that the author himself has used and explored in his work on blade vibrations are now laid out in historical terms, thus making clear to us how the many clever innovators that we have to thank for our firm grip on the technology of our subject achieved their results. The contributions of Euler, Lagrange, Rayleigh, and Dunkerley to the history of this subject are now set before us in fresh, historical terms, rather than in the familiar terms with which we are familiar. Here, these authors appear as real people for the first time, in the perspective of their subject. Euler with his thirteen children, going blind in St. Petersburg; Lagrange, too worried after publishing his timeless *Mechanique Analytique* to even open the copy on his desk for two years, amid the revolution which swirled below in the streets of Paris; and Rayleigh, in the blissful isolation of a honeymoon cruise up the Nile River, writing the *Theory of Sound*. Now at last for us these people have real, human faces.

In quick succession Rao now moves through the graphical methods of the late 19th and early 20th century until need again outstrips ability. Here it is the hands of Holzer which introduce numerical methods to address situations where formulas only lead to intractable mathematics. We begin to see that future the world of technology will exist in digital terms, and not formula terms. Jeffcott studied the vexed “critical speed” problem of shaft dynamics, by making the first rigorous experimental study, and then by using these results during his analysis of the problem. He found that this was a forced vibration problem, not an instability problem as some before him had claimed. Rao next considers the development and the contributions of the computer. He considers in parallel the birth of matrix methods, and of the Prohl–Myklestad method, as preparations through design for the solution of problems relating to the unbalance response and balancing of real rotors. Thus this fundamental question of rotordynamics had waited sixty years before it was explained by Jeffcott’s contribution, and then it waited a further forty years, through the innovations and squabbles of the 1970s, until a full and accepted solution for unbalance response was finally reached.

This of course says nothing about the other great problem of rotordynamics which emerged in the 1920s, that of rotor instability. Rao’s book treats both bearing instability and shaft hysteretic instability, and considers the impact of this and other questions of shaft dynamics, such as misalignment and asymmetry of the rotor. He makes it clear why almost a hundred years had to elapse before a complete solution for the shaft problem was reached. To complete his work, Rao addresses the much-awaited introduction of the Finite Element method into both rotor-dynamics and bearing dynamics, and then turns to another topic for which he is well prepared: the problems of blade and disk dynamics. Rao describes from first-hand knowledge how the blade vibration problem with its many geometrical complexities was investigated. He describes the evolution of methods for single airfoil analysis with discrete bending-bending-torsion approaches, and eventually through to finite element approaches, where with the evolution of greater computer capabilities it became

possible to study both the steady stresses and the dynamic stresses in blades, blade groups, and today in complete blade-disk assemblies.

We may be confident that with this history, as part of the larger history of the Theory of Machines and Mechanisms that Professor Marco Ceccarelli has so worthily undertaken as editor, will do much to set the record straight on the origins of rotor-dynamics within the historical perspective of machinery development. This book, and other works in this series, will be received and read with relish by savants and students alike, that is, by all who seek knowledge of the background to their art. With the writing of this book, Dr Rao has secured for himself yet again a place in the history of engineering, as much as his technical work in this subject has earned him a place in its science. All who practice this fine art must now feel indebted to Dr Rao for this impressive work of documentation and perspective.

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# Preface

It has been an eventual 20th century that transformed the mankind in the rapid development and usage of rotating machinery in various industrial sectors. Its all due to de Laval and Parsons inventions of impulse and reaction turbines who could bring the dream of Hero of Alexandria alive in 2100 years. This book is written to retrace the steps of history.

While the book traces the events leading to Laval and Parsons Turbines, the emphasis is on rotor and blade dynamics aspects that pushed these turbines to limits in the last century. The tabular and graphical methods developed in precomputer era have taken different form in the last 50 years through finite element methods. The methods evolved in the last century are discussed in detail to help the modern day designers and researchers.

Man has become mobile with the ending of last ice age 15000 years ago and has since been looking for doing daily chores in an easier manner. He invented the wheel for this purpose and put it use nearly 5000 years ago in the form of a Potter's wheel, then to transportation and as a grinding wheel in about 3000 years of time. This wheel is fundamental to rotating machinery as briefly presented in Chapter 1.

There was no science to take this development further, the first organized thinking before medieval period came from Aristotle about 2350 years ago. Archimedes in Alexandria about 2250 years ago has put forward some realistic and sound ideas on science; in fact in his period, Hero made the first workable reaction steam turbine. This was the first machine man attempted but could not take it to real usage because of lack of science. Chapter 2 briefly discusses these.

Without much science man has used wheel in the form of Water wheels and Wind mills to reduce the burden of grinding food grains and smithy. These are discussed briefly in Chapters 3 and 4.

The need for science was felt more and more in understanding structures and despite strict religious practices; there was renaissance in scientific thinking as by Leonardo da Vinci that led to scientific revolution with Nicholas Copernicus announcing heliocentric theory. Events moved fast with the invention of Calculus by Newton and Leibniz and beam structures received maximum attention three cen-

turies ago. The rotor owes its knowledge base from the beam theories in this scientific revolution period. These aspects were discussed in Chapter 5.

While the science is taking good shape, engineers were looking forward to develop machines particularly removing water from mines. Otto von Guericke discovered vacuum in this period and renaissance engineers beginning from Denis Papin developed “atmospheric engines” or “fire engines” for this purpose. These developments were briefly discussed in Chapter 6.

With the discovery of Latent Heat by Joseph Black, the stage has been set to James Watt for producing a reciprocating steam engine which ushered in industrial revolution more than 200 years ago. Chapter 7 briefly discusses these aspects.

The reciprocating engine was never the dream of mankind and with Laval and Parsons Inventions, the reciprocating steam engine gave way to turbomachines which has rotors and mounted blades; this engine was considered “Vibration Free”. This is the beginning of rotor dynamics just over a century ago. Chapter 8 briefly discusses these aspects.

The rotor and blade dynamics depended heavily on science and the fundamentals of elasticity are first discussed in Chapter 9. The elasticity equations are not amenable for a solution that involves coupled partial differential equations with 15 unknowns. Therefore energy methods were developed using Calculus of Variations from Newton which are fundamental to derivation of modern finite element methods. The energy methods were discussed in Chapter 10. We also discuss here some design examples adopted from Rayleigh’s principle as adopted over a century ago in industry. Hamilton’s principle which is the most general principle in Dynamics is discussed in this chapter with examples that cannot be addressed with finite elements even today.

When the turbomachinery were invented and their application became ubiquitous with the dynamo, the industry needed methods to determine critical speeds of rotors and blades. Since the methods of elasticity were not suitable without computers, initially strength of materials approach was developed and numerical methods using graphical or tabular form were adopted. These methods were discussed in Chapter 11.

Matrix methods were adopted in the mid 20th century and discussed in Chapter 12. With the advent of computer era the finite element method made rapid strides and they are discussed in Chapter 13 for beams, plates and shells. Nonlinear contact element methods required for analysis of bladed-disks were also discussed here. Typical application of commercial finite element codes for a turbomachine blade as practiced today is also discussed.

Rotor dynamics deals with rotating structures and the way in which these principles were developed from Jeffcott analysis in 1918 to handle various special aspects on fluid film supports, instabilities, gyroscopic effects, etc., are presented in Chapter 14. These methods in transfer matrix form are discussed in Chapter 15. The application of finite element methods for rotor dynamics is discussed in Chapter 16. Solid model rotor dynamics a recent development is also discussed in this chapter.

Blades are mounted on rotors and they form important rotor dynamics in turbomachinery. The developments in blades and bladed-disks are discussed in Chapter 17.

With rapid development and deployment of commercial softwares, simulation and lifting of rotating machinery components has become a standard design practice. Recently the industrial designs demand optimum solutions in weight as well as life and this practice is becoming a standard norm. These methods were briefly discussed in Chapters 18 and 19.

The information presented in this book will be useful to a young researcher and engineer in industry and educational institutions engaged in rotor and blade dynamics work in understanding the past and the present developments and what is expected in future. Faculty and industry engineers can have a broad perspective in this field in formulating their developmental plans.

It should be mentioned here that it is a near impossible task to do justice for such a historical survey in rotating machinery. The basics of rotating machinery are in Thermodynamics, Compressible Flow, Heat Transfer, and Material Science besides instrumentation and controls, and we have restricted to rotor and blade dynamics. The excitation comes from Flow path interference in a stage with flutter in some cases; this has not been included in this book. In rotor dynamics, magnetic bearings are a topic and this has also been not included here. Condition monitoring of rotating machinery also has its roots in rotor dynamics and not included. Also despite a wide search, some significant developments could have easily escaped my attention and my apologies for the same.

I sincerely hope that rotor and blade dynamics community will enjoy reading this material.

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