

AN INTRODUCTION TO PLASMA ASTROPHYSICS  
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# AN INTRODUCTION TO PLASMA ASTROPHYSICS AND MAGNETOHYDRODYNAMICS

*by*

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# Preface

This book aims to give a basic introduction to plasma physics with an emphasis on Magnetohydrodynamics (MHD for short). It has grown out of the lecture notes that I have been teaching at the K.U.Leuven for the last fifteen years to third and fourth year undergraduate students in applied mathematics and physics. For students at the K.U.Leuven this course is their first encounter with plasma physics. Hence, its level is elementary. Since both students in mathematics and physics can take this course, both mathematical integrity and physical intuition are pursued.

The content of this book might not be what is routinely expected from an introductory book on plasma physics. Often introductory courses present various approximate plasma models with minimal discussion of the underlying theoretical foundations and the simplifying assumptions. In contrast, I have decided to give special attention to fundamental concepts and first principles and to limit the discussion of plasma models to a large extent to Magnetohydrodynamics (MHD). The equations of MHD are derived from first principles starting from the Boltzmann equations for the distribution functions in phase space of the various species. It is clear that this approach is not the best way to economically present the equations of MHD. However, it has the considerable advantage that it allows to show where and how microphysics has been removed and to determine the limitations and the domain of validity of MHD. I have found out that an elementary level and a concern with fundamental concepts and first principles are not mutually exclusive, but go together very well. The reader will not end up with the wrong impression that MHD covers all of plasma physics. He/she will be fully aware that plasmas can show behaviour that cannot be captured with MHD. In that sense the course is also an invitation to the interested reader to go beyond classic MHD and discover many fascinating plasma behaviour that is not dealt with here.

I decided to set this course in the framework of solar and space plasma physics and astrophysics. Hence, the name of the course: "Introduction to Plasma Astrophysics and Magnetohydrodynamics". The fully and partially ionized plasmas that are the central focus of solar and space physics are related on a fundamental level to laboratory plasma physics, which directly investigates basic plasma physical processes, and to astrophysics, a discipline that relies heavily on understanding the physics of the plasma state. Although solar physics is the framework in which I want to set my course, I have refrained myself from concentrating on a description of a large number of plasma physics phenomena in solar physics and astrophysics. The first chapter gives an incomplete and short overview of plasma physics phenomena in solar physics and astrophysics. The last chapter is the only chapter that deals exclusively with solar physics. It discusses the solar wind in the context of hydrodynamics and MHD.

The amount of mathematics and physics required for using this book is limited. A knowl-

edge of vector calculus, real calculus and electromagnetic theory are the modest prerequisites from mathematics and physics. The exercises take a special place in the course and in this book. We all learn best the things that we have discovered for ourselves. Hence, in stead of being very detailed in the derivation of the equations and results, I have taken the relaxed attitude to be economical on intermediate results and steps when these are straightforward. A first class of exercises invites the reader to fill in gaps in the often long derivation of equations. There is no need for the reader to memorize the different steps required for obtaining a given equation or result, but he/she should have gone through this straightforward mathematics at least once. Also, it is difficult, if not impossible, for anyone to learn a subject purely by reading about it. Applying the information to specific problems and thereby being encouraged to think about what has been read, is essential in the learning process. A second class of exercises tries to invite the reader to just do that. The exercises form a major part of this book. In the Belgian educational system students have to take exams. The exam for the material covered in this book consists of solving the exercises and explaining the solutions by using the notes the students have prepared themselves.

A short and elementary book on “Plasma Astrophysics and Magnetohydrodynamics” cannot focus on recent research results which require a deep understanding of the subject. Even at this elementary level I feel that the insights and the interpretations that I try to convey in this book, are influenced and shaped to a large extent by the scientific collaborations and discussions I have had over the years with numerous colleagues and friends including in particular L. Mestel, Z. Sedlacek, A.D.M. Walker, J.P. Goedbloed, F. Verheest, E.R. Priest, J.V. Hollweg, W. Kerner, B. R. Roberts, M. Ruderman, T. Sakurai, K. Tsinganos, A. W. Hood, Y. Voitenko, D. Van Eester, S. Poedts, R. Erdelyi, R. van der Linden and R. Keppens. In addition to being instructive, it was fun. Thank you. My gratitude also goes to A. De Groof for her help in preparing this book and to P. Charbonneau for providing me with ps-files of figures of his unpublished class notes on “Large Scale Dynamics of the Solar Wind”. It is a pleasure to thank Kluwer Academic Publishers for giving me the opportunity to publish my class notes in the Astrophysics and Space Science Library Series. I have benefitted from several good books on plasma physics, magnetohydrodynamics and solar physics. Those that I like the best are listed at the end of the introductory chapter under references. These are, with one or two exceptions, the only references given in this book. The material covered in this book is at the basic elementary level and owned as it were by the community. It should be clear that nothing of the work described in this book is my own. The book is based on the work by the pioneering giants J.C. Maxwell, L. Boltzmann, I. Langmuir, J. Larmor, H. Alfvén and E. Parker.

The students who have taken this course over the years, probably do not realize it, but I have benefitted a lot from them. Their criticism and questions have helped me shaping the notes in their present form. The fact that several of these former students are pursuing scientific and academic careers in which mathematical modelling of plasmas and MHD still play a prominent role, is reassuring to me. It has not all been in vain. Most of the students who have taken the course, do not use its content in their daily professional life. For them I dare hope that the course has contributed to their scientific training by learning them how mathematical modelling and physical intuition and interpretation can go hand in hand. The importance of mathematical modelling in this context must be stressed. Even when the mathematical description has been simplified by replacing a description using the Boltzmann equations for the distribution functions in phase space of the various species, with a description based on MHD, mathematical modelling is often essential. The full nonlinear equations of

MHD are so complicated, that they often need to be approximated drastically by focussing on the dominant physical mechanisms in a any particular situation. When solutions for simple situations are known, more and more effects may be added to make the model more realistic. It is my hope that this book may help students at the K.U.Leuven and elsewhere to appreciate the intriguing and complicated behaviour of plasmas and to appreciate the power of mathematical modelling as a tool for exploring and understanding this complicated behaviour.

March 2003

Marcel Goossens

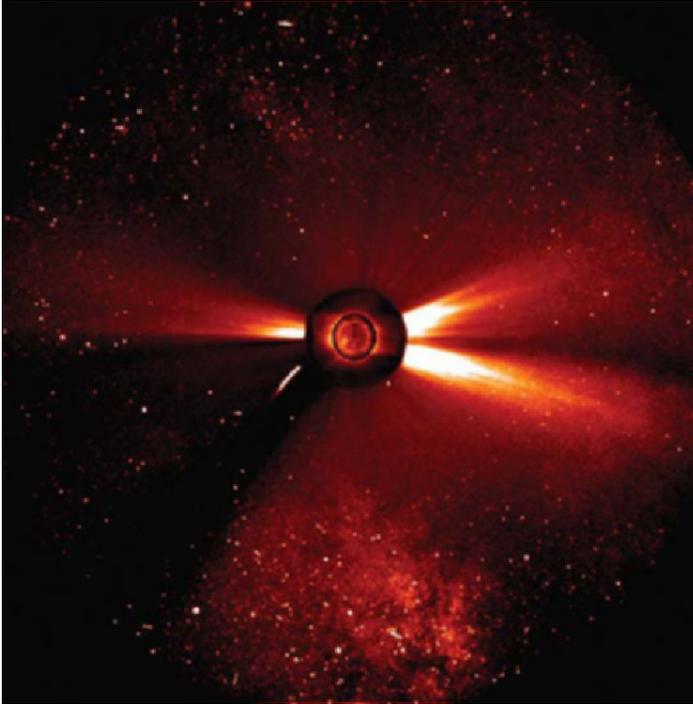


Figure 1: *Cover Illustration.* (Courtesy of SOHO (ESA & NASA.))

### Cover illustration

This cover illustration is a composite of

- EIT EUV image taken in the Fe XV line at 284Å showing the corona above the disk at a temperature of about 2-2.5 million K (innermost image)
- UVCS image showing the Sun's outer atmosphere as it appears in ultraviolet light emitted by electrically charged oxygen (O VI) flowing away from the Sun to form the solar wind (middle region), and
- Image of the extended white light corona as recorded by the outer LASCO coragraph (C3) on 23 December 1996.

The field of view of this instrument encompasses 32 diameters of the Sun. To put this in perspective, the diameter of this image is 45 million kilometers at the distance of the Sun, or half of the diameter of the orbit of Mercury. During that time of the year, the Sun is located in the constellation Sagittarius. The center of the Milky Way is visible, as well as the dark interstellar dust rift, which stretches from the south to the north. Three prominent streamers can be seen (two at the West and one at the East limb). This image also shows

Comet SOHO-6 (elongated streak at about 7:30 hours), one of several tens of sun-grazing comets discovered so far by LASCO. It eventually plunged into the Sun.

This composite image can be found at <http://sohowww.nascom.nasa.gov/gallery/LASCO/>  
(Courtesy of SOHO (ESA & NASA.))