Advanced Integration Theory

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Preface

Since about 1915 integration theory has consisted of two separate branches: the abstract theory required by probabilists and the theory, preferred by analysts, that combines integration and topology. As long as the underlying topological space is reasonably nice (e.g., locally compact with countable basis) the abstract theory and the topological theory yield the same results, but for more complicated spaces the topological theory gives stronger results than those provided by the abstract theory. The possibility of resolving this split fascinated us, and it was one of the reasons for writing this book.

The unification of the abstract theory and the topological theory is achieved by using new definitions in the abstract theory. The integral in this book is defined in such a way that it coincides in the case of Radon measures on Hausdorff spaces with the usual definition in the literature. As a consequence, our integral can differ in the classical case. Our integral, however, is more inclusive. It was defined in the book "*C. Constantinescu and K. Weber* (in collaboration with *A. Sontag*), Integration Theory: Measure and Integral, John Wiley & Sons, 1985", where the chief goal was to establish its basic properties and to discuss the relation between the new and the old definition of the integral. In the present book which is independent of the above quoted book, we pursue a pragmatic point of view: we define the integral without presenting a detailed rationale for the definition that is given, and develop in the sequel the integration theory.

It is our belief that certain important topics in integration theory are best understood in the context of vector lattices. Accordingly, we begin with a thorough study of the theory of vector lattices. We present then the theory of L^p–spaces (for $0 < p \leq \infty$), including duality, and the theory of spaces of real–valued measures on a ring of sets, including the Radon–Nikodym Theorem. Vector lattices are not merely a passive backdrop for this presentation. Rather, the vector–lattice setting suggests important concepts and provides powerful techniques.

The book concludes with an extensive chapter on classical integration theory on $\mathbb{R}$. Although we have chosen a rather abstract and very general framework for integration theory, we are convinced that the classical theory represents a very important part of integration theory and should not be neglected in a book such as this. We also believe that the beauty of the classical theory is enhanced when this theory is viewed through the abstract theory.

Exercises provided for each section augment the treatment in the text. A number of exercises are new. Some others stem from the existing literature, although we do not provide references. We would like to make clear that in many exercises significant results have made their way into the book, results for which there was no room in the text. Many such exercises are of interest in themselves. To enable the reader to make effective use of the exercises, sketches of solutions are often included. For additional remarks describing the role played in this book by the exercises we ask the reader to consult the chapter entitled "Suggestions to the Reader".

Certain points of view that influenced this book should be mentioned. One use we hope the book will find is as a text for a graduate course or seminar or for self–study. For this and other reasons the book is essentially self–contained: the only prerequisite is familiarity with elementary real analysis. In other words, we have tried to write a book so that someone who knows no integration theory (but has the prerequisite real analysis and is moderately perseverant) can read and learn integration therory by doing so. In particular, we have purposely chosen not to gloss over technical details. Such details may occasionally dim the appeal of the story, but they are nevertheless necessary. After all, what is trivial to one who knows and understands a theory is not always so to one who is just learning it. Such factors contribute to the length of the book.

It is also expected that many users of the book will use it as a reference in their own research, whether in integration theory or in other fields. It is our hope that such users not only will find the information they need but also will use the book as a conveniently citable, and readily decipherable source.

The text is organized in the definition–theorem–proof format that is by now familiar. We find that this organizational format when strictly adhered to, simplifies the task both for the beginner who seeks to learn by reading the book and the expert who seeks to refresh his or her memory or to find an appropriate theorem to quote. We hope that our use of this format has not altogether hidden the fascination that prompted our writing of the book. Moreover, it need not preclude narration and commentary. We have attempted to describe the ideas of the proofs, not just the formal details. Commentary has been used to signal important features of the definition–theorem–proof landscape. We hope that readers can thereby see in advance roughly where they are headed (and where possibly the course to be pursued).

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