Linear and Nonlinear Control of Small-Scale Unmanned Helicopters

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to my parents, Anastasios and Alexandra,
and to my sister, Martha *(IAR)*

to my mother, Stella
to my brother, Anthony, and
to my children, Stella and Panos *(KPV)*

Preface

Unmanned Aerial Vehicles (UAVs) or Unmanned Aircraft Systems (UAS), a term preferred by the U.S. Department of Defense, have seen unprecedented levels of growth over the last decade. Even though UAVs have been mainly used for military applications, there is a considerable and increasing interest for civilian applications. It is not an exaggeration to consider that as the technology matures, as small-scale UAVs become cost-effective with proven reliability and safety, and as the roadmap to integrating UAS into the National Airspace System (NAS) progresses, civilian applications will dominate the field. It is postulated that UAVs will be used in the future extensively for environmental monitoring, forest protection, wildfire detection, traffic monitoring, building, power line and bridge inspection, emergency response, crime prevention, search and rescue, mapping, surveillance, reconnaissance, border patrol, to name several applications.

From all classes of UAVs, unmanned rotorcraft, and in particular unmanned helicopters, have advantages over fixed-wing UAVs because they take-off and land vertically, they do not require a runway, and they have the ability to hover and fly in (very) low altitudes. It is reasonable to assume that light-weight (<150 Kgr) and small-scale (<50 Kgr) helicopters will be the first ones to be allowed to fly in civilian airspace. Such helicopters, though, still retain the flight characteristics and physical principles of their full-scale counterparts. In addition, they are naturally more agile and dexterous compared to full-scale helicopters. Their flight capabilities, reduced size and cost have recently monopolized the attention of the UAV research community as they are preferred for a wide spectrum of applications. However, helicopters are highly unstable, nonlinear and coupled underactuated systems, and controller design for such systems is a rather challenging problem.

The problem of designing autonomous flight controllers for small-scale helicopters is equally challenging, and the flight controller design problem is tightly connected with the helicopter modeling. Helicopter dynamics may be represented by both linear and nonlinear models of ordinary differential equations. Typically, the validity of the linear models is restricted in a certain region around a specific operating point, while nonlinear models provide a global description of the helicopter dynamics.

Therefore, it is the goal of this book to present a rather comprehensive and well justified analysis for designing (autonomous) controllers for small-scale unmanned helicopters, and then present details on how to design MIMO linear, continuous and discrete time nonlinear controllers for such helicopters guarantying stability. The controllers objective is for the helicopter to autonomously track predefined position (or velocity) and heading reference trajectories, evaluating their performance using *X-Plane*, a realistic and commercially available flight simulator.

However, as in most control applications, the helicopter model that is used for controller design purposes is just an approximation of the actual nonlinear helicopter dynamics. Thus, in order to develop a generic flight control system, which applies to most standard small-scale helicopter platforms, the designer must successfully solve three intermediate tasks: (i) Derive the structure and the order of a parametric dynamic model that best describes the helicopter motion; any derived parametric model should provide a physically meaningful dynamic description for a large family of small-scale helicopters. (ii) After the parametric helicopter model is derived, one must determine a nominal feedback control law such that the helicopter tracks a predefined reference trajectory. The design should guarantee that the control inputs remain bounded while the helicopter tracks the reference trajectory. (iii) Given a specific helicopter, one must determine which is the best methodology to accurately extract the values of the parametric model that will be used to implement the linear/nonlinear controllers.

The reader is introduced to the controller design challenges in a step-by-step way. At first, an analytical derivation of the helicopter's kinematic equations of motion is presented with the helicopter treated as a rigid body, followed by a simplified model of the main rotor dynamics that encapsulates the coupling effects between the fuselage motion and the main rotor of the helicopter. Next, the reader is introduced to linear controller designs based on a frequency domain identification method that is used for the extraction of low order linear helicopter models. Then, the focus shifts to controller designs based on the nonlinear helicopter model. The design approach is very rigorous and detailed following the backstepping methodology for systems in feedback form. Continuous and discrete time nonlinear controllers are presented, and a simple Recursive Least Squares (RLS) method is employed to identify the parameters of the discrete nonlinear helicopter model. It is also demonstrated how a Takagi–Sugeno fuzzy system may improve the time domain identification results of the RLS algorithm. An extensive comparison and evaluation of all controller designs is also included in this book. The rationale for such a study is to pave the way for a rather comprehensive performance evaluation of controllers, and at the same time justify and support the chosen methodologies.

The reader is expected to have knowledge of modern control theory as the minimum prerequisite to follow the book, as well as an understanding of fundamentals of kinematics and dynamics.

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Contents

Acronyms

ACAH	Attitude-Command Attitude-Hold
CG	Center of Gravity
CIFER	Comprehensive Identification from FrEquency Responses
DOF	Degrees Of Freedom
FAA	Federal Aviation Administration
GAS	Globally Asymptotically Stable
LTI	Linear Time Invariant
MIMO	Multiple-Input Multiple-Output
MTOW	Maximum Take-Off Weight
NN	Neural Networks
PD	Proportional Derivative
PID	Proportional Integral Derivative
PBH	Popov–Belevitch–Hautus
RC	Radio Controlled
RLS	Recursive Least Squares
SISO	Single-Input Single-Output
SMD	Spring Mass Damper
TPP	Tip-Path-Plane
UDP	User Datagram Protocol
UGAS	Uniformly Globally Asymptotically Stable
UGB	Uniformly Globally Bounded

Symbols

a	TPP state vector
a, b	Longitudinal and lateral tilt angles of the TPP
$\bar{a}, \bar{b}$	Applied flapping angles
A, B	Matrices of the helicopter's linear state space model
A_b, B_a	TPP dynamics cross coupling terms
$A_{\text{ll}}, B_{\text{ll}}, A_{\text{yh}}, B_{\text{yh}}, D_{\text{yh}}$	Longitudinal–lateral and yaw–heave subsystems state space matrices
$\mathcal{A}_{\text{ll}}, \mathcal{B}_{\text{ll}}, \mathcal{A}_{\text{yh}}, \mathcal{B}_{\text{yh}}, \mathcal{D}_{\text{yh}}$	Longitudinal–lateral and yaw–heave subsystems (including the position and yaw integral error) state space matrices
$A_{\text{ll}}^{\text{cl}}, A_{\text{yh}}^{\text{cl}}$	Longitudinal–lateral and yaw–heave subsystems closed loop matrices
$\mathcal{A}_{\text{ll}}^{\text{cl}}, \mathcal{A}_{\text{yh}}^{\text{cl}}$	Longitudinal–lateral and yaw–heave subsystems (including the position and yaw integral error) closed loop matrices
$A_{\text{ll}}^{\text{fb}}$	Identical matrix to A_{ll} neglecting the X_a and Y_b stability derivatives
$A_{\text{lon}}, A_{\text{lat}}$	Constants relating the cyclic commands with the cyclic pitch angles of the blade
c_b	Blade's chord
C_d	Drag constant
$C_{l\alpha}$	Airfoil's lift curve slope
C^M, D^M	Constants associated with the anti-torque Q_M
D, K, F	Damping, stiffness and forcing function matrices of the TPP dynamics
dD	Drag produced by the blade element
dF_a	Aerodynamic force acting on the blade element
dF_c	Centrifugal force of the blade element
dF_i	Inertia force of the blade element due to flapping
dL	Incremental lift produced by the blade element
dm	Elementary mass of the helicopter

$d_x^f, d_y^f, d_z^f, d_y^{\mathrm{vf}}, d_z^{\mathrm{hs}}$	Parameters that depend on the air density, the geometry of the fuselage, the vertical fin and the horizontal stabilizer
$e_p^B = p^B - p_r^B$	Position error expressed in the body-fixed frame
$e_{\mathrm{ll}}, e_{\mathrm{yh}}$	Longitudinal–lateral and yaw–heave error subsystems state vectors
e_x	Error of the variable x minus its desired value x_d
$\vec{f}$	Total force vector acting on the fuselage
$f^B = (X\ Y\ Z)^T$	Total force, acting on the fuselage, with respect to the body-fixed frame
f_d^B	Drag force vector with respect to the body-fixed frame
F^B	External wrench acting on the helicopter with respect to the body-fixed frame
F_a	Fuzzy set representing the linguistic variable $\bar{a}$
$\mathcal{F}_B = \{O_B, \vec{i}_B, \vec{j}_B, \vec{k}_B\}$	Body-fixed frame
$\mathcal{F}_h = \{O_h, \vec{i}_h, \vec{j}_h, \vec{k}_h\}$	Hub frame
$\mathcal{F}_I = \{O_I, \vec{i}_I, \vec{j}_I, \vec{k}_I\}$	Inertial Earth-fixed frame
g	Gravitational constant
$\vec{h}_M$	Position vector of the main rotor from the CG
$h_M^B = (x_m\ y_m\ z_m)^T$	Coordinates of the main rotor shaft with respect to the body-fixed frame
$\vec{h}_T$	Position vector of the tail rotor from the CG
$h_T^B = (x_t\ y_t\ z_t)^T$	Coordinates of the tail rotor shaft with respect to the body-fixed frame
$\vec{H}$	Angular momentum vector
$H^B = (h_x\ h_y\ h_z)^T$	Angular momentum with respect to the body-fixed frame
$\mathcal{I}$	Inertia matrix
$\mathcal{I}_b$	Inertia of the blade
$\mathcal{I}_{xx}, \mathcal{I}_{yy}, \mathcal{I}_{zz}$	Moments of inertia
$\mathcal{I}_{xy}, \mathcal{I}_{yx}, \mathcal{I}_{xz}, \mathcal{I}_{zx}, \mathcal{I}_{yz}, \mathcal{I}_{zy}$	Products of inertia
$J(\Pi)$	Cost function (dependent on the parameter vector Π)
$J_{\mathrm{ll}}, J_{\mathrm{yh}}$	Longitudinal–lateral and yaw–heave error subsystems performance indexes
$\mathcal{J}_{\mathrm{ll}}, \mathcal{J}_{\mathrm{yh}}$	Longitudinal–lateral and yaw–heave error subsystems (including the position and yaw integral error) performance indexes
K_β	Stiffness of the rotor hub
L, M	Thresholds of the saturation function σ
L_b, M_a	Stability derivatives of the pitch and roll moments
m	Total mass of the helicopter
m_b	Mass per unit length of the blade
N	Number of samples
$\vec{p}$	Position vector of the helicopter CG
$p^I = (p_x^I\ p_y^I\ p_z^I)^T$	Position with respect to the inertial frame

Q_M	Main rotor reaction torque
$Q_{\text{ll}}, R_{\text{ll}}, Q_{\text{yh}}, R_{\text{yh}}$	Longitudinal–lateral and yaw–heave error subsystems LQR design matrices
$\mathcal{Q}_{\text{ll}}, \mathcal{R}_{\text{ll}}, \mathcal{Q}_{\text{yh}}, \mathcal{R}_{\text{yh}}$	Longitudinal–lateral and yaw–heave error subsystems (including the position and yaw integral error) LQR design matrices
R	Rotation matrix
R_b	Blade's radius
R_l	Total number of fuzzy rules
$\mathcal{R}_{xx}(\tau)$	Auto correlation of the signal $x(t)$
$\mathcal{R}_{xy}(\tau)$	Cross correlation of the signals $x(t)$ and $y(t)$
$S_{xx}(j\omega)$	Auto spectral density of the signal $x(t)$
$S_{xy}(j\omega)$	Cross spectral density of the signals $x(t)$ and $y(t)$
$\hat{S}_{xx}(j\omega)$	Discrete estimate of the auto spectral density S_{xx}
$\hat{S}_{xy}(j\omega)$	Discrete estimate of the cross spectral density S_{xy}
$SE(3)$	Special Euclidean group
$SO(3)$	Special Orthogonal group of order 3
T_b	Period of the flapping motion
$T_{\max}$	Maximum period of interest
$\vec{T}_M$	Main rotor thrust vector
T_M	Magnitude of the main rotor thrust vector
$T_M^B = (X_M \; Y_M \; Z_M)^T$	Components of the main rotor thrust vector with respect to the body-fixed frame
T_{rec}	Length of flight record
T_s	Sampling period
$\vec{T}_T$	Tail rotor thrust vector
T_T	Magnitude of the tail rotor thrust vector
$T_T^B = (0 \; Y_T \; 0)^T$	Components of the tail rotor thrust vector with respect to the body-fixed frame
$\bar{T}_M, \bar{T}_T$	Servo outputs
u_c	Helicopter's control input vector
u_c^d	Desired control input vector
u_c^{ss}	Steady state control input vector
$u_{\text{ll}}, u_{\text{yh}}$	Longitudinal–lateral and yaw–heave subsystems control input vectors
u_i	Inflow velocity
u_l	Control law based on the helicopter's linear model
u_n	Control law based on the helicopter's nonlinear model
u_{col}	Main rotor collective control command
u_{lat}	Main rotor lateral cyclic control command
u_{lon}	Main rotor longitudinal cyclic control command
u_{ped}	Tail rotor pedal control command
U	Resultant air velocity of each blade element
U_P	Perpendicular velocity of the blade element
U_R	Radial velocity of the blade element

U_T	Tangential velocity of the blade element
$\vec{v}$	Linear velocity vector of the fuselage
$v^B = (u\ v\ w)^T$	Linear velocity with respect to the body-fixed frame
$v^I = (v_x^I\ v_y^I\ v_z^I)^T$	Linear velocity with respect to the inertial frame
$v_a^B = (v_{a,x}^B\ v_{a,y}^B\ v_{a,z}^B)^T$	Relative wind velocity vector with respect to the body-fixed frame
v_w^B	Wind velocity in the body-fixed frame coordinates
V_∞	Free stream velocity
$W(\omega_i)$	Weighting matrix of the cost function $J(\Pi)$
x	Actual helicopter state
x_d	Desired state vector
x_l	State vector of the helicopter's linear model
x_n	State vector of the helicopter's nonlinear model
x_{ss}	Steady state response
$x_{\mathrm{ll}}, x_{\mathrm{yh}}$	Longitudinal–lateral and yaw–heave subsystems state vectors
$x_{\mathrm{ll}}^d, x_{\mathrm{yh}}^d$	Longitudinal–lateral and yaw–heave subsystems desired state vectors
x_h, y_h, z_h	Coordinates of the tip of the blade with respect to the hub frame
x_m, y_m, z_m	Distances of each elementary mass from the CG
$X(j\omega)$	Fourier transform of the signal $x(t)$
X_a, Y_b	Linear velocity stability derivatives
y	Helicopter's output vector
y_m	Vector of the helicopter's available measurements
y_r	Vector of the helicopter's reference trajectories
$y_{\mathrm{ll}}, y_{\mathrm{yh}}$	Longitudinal–lateral and yaw–heave subsystems output vectors
$y_{\mathrm{ll}}^m, y_{\mathrm{yh}}^m$	Longitudinal–lateral and yaw–heave subsystems measurement vectors
$y_{\mathrm{ll}}^r, y_{\mathrm{yh}}^r$	Longitudinal–lateral and yaw–heave subsystems reference output vectors
$Y_{\mathrm{ll}}, Y_{\mathrm{yh}}$	Longitudinal–lateral and yaw–heave error subsystems output vectors
$Y_{\mathrm{ll}}^m, Y_{\mathrm{yh}}^m$	Longitudinal–lateral and yaw–heave error subsystems measurement vectors
$\mathcal{Y}_{\mathrm{ll}}, \mathcal{Y}_{\mathrm{yh}}$	Longitudinal–lateral and yaw–heave subsystems (including the position and yaw integral error) output vectors
$\mathcal{Y}_{\mathrm{ll}}^m, \mathcal{Y}_{\mathrm{yh}}^m$	Longitudinal–lateral and yaw–heave subsystems (including the position and yaw integral error) measurement vectors

Greek Symbols

α_b	Blade's angle of attack
α_{hb}	Angle of the free stream velocity with respect to the hub plane
β	Flapping angle of the blade
$\beta_0, \beta_{1s}, \beta_{1c}$	First harmonic terms of the TPP
γ	Lock number
γ_{xy}^2	Coherence function of the signals $x(t)$ and $y(t)$
δ	Perturbed value of a variable
$\epsilon(\omega, \Pi)$	Vector of the frequency response magnitude and phase errors
$\varepsilon_{\text{ll}}, \varepsilon_{\text{yh}}$	Longitudinal–lateral and yaw–heave error subsystems (including the position and yaw integral error) state vectors
ζ	Feathering angle of the blade
ζ_0	Collective pitch of the blade
ζ_{1c}, ζ_{1s}	Cyclic pitch angles
θ	Pitch angle
$\Theta = (\phi\ \theta\ \psi)^T$	Orientation angles vector
λ_β	Flapping frequency ratio
μ	Rotor's advance ratio
μ	Membership function
ξ	Lead-Lagging angle of the blade
Π	Parameter vector
Π_n	Parameters vector of the linear helicopter model
Π_n	Parameters vector of the nonlinear helicopter model
Π_{TS}	Parameter vector of the Takagi–Sugeno fuzzy system
$\tilde{\Pi}$	Estimate of the parameter vector
ρ_a	Air density
$\rho_d = (\rho_{d,1}\ \rho_{d,2}\ \rho_{d,3})^T$	Desired direction of the vector ρ_3
ρ_i	ith column vector of the rotation matrix
$\rho i, j$	Element of the jth row and ith column of the rotation matrix

$\varrho = (\varrho_1\ \varrho_2)^T$	Reduced direction vector
$\varrho_d = (\varrho_{d,1}\ \varrho_{d,2})^T$	Desired reduced direction vector
σ	Saturation function
Σ	Vector with saturation functions as its components
τ	Time delay
$\vec{\tau}$	Total torque vector acting on the fuselage
$\tau^B = (L\ M\ N)^T$	Total torque, acting on the fuselage, with respect to the body-fixed frame
$\vec{\tau}_\beta$	Main rotor moments due to the hub stiffness
$\tau_\beta^B = (L_\beta\ M_\beta\ N_\beta)^T$	Components of $\vec{\tau}_\beta$ with respect to the body-fixed frame
τ_d^B	Drag moment vector with respect to the body-fixed frame
$\vec{\tau}_M$	Main rotor moment vector
$\tau_M^B = (L_M\ M_M\ N_M)^T$	Main rotor moment vector with respect to the body-fixed frame
$\vec{\tau}_T$	Tail rotor moment vector
τ_f	Main rotor time constant
$\vec{\tau}_Q$	Moment vector due to hub stiffness and main rotor anti-torque
$\tau_Q^B = (R_M\ M_M\ N_M)^T$	Components of $\vec{\tau}_Q$ in the body-fixed frame
ϕ	Roll angle
ϕ_b	Blade's induced angle of attack
ψ	Yaw angle
ψ_b	Blade's azimuthal angle
$\Psi_3(\Theta)$	Third row of the matrix $\Psi(\Theta)$
ω	Angular frequency
$\vec{\omega}$	Angular velocity of the fuselage
$\omega^B = (p\ q\ r)^T$	Angular velocity with respect to the body-fixed frame
ω_i	Frequency point
$\omega_{\min}, \omega_{\max}$	Minimum and maximum frequency of interest
ω_n	Natural frequency
Ω	Angular velocity of the helicopter blades
Ω_s	Frequency resolution

Subscripts and Superscripts

ll	Longitudinal–lateral subsystem
yh	Yaw–heave subsystem
d	Desired value of a variable
m	Indicates a vector or a matrix associated with the measured output
r	Reference value of a parameter
ss	Steady state value of a state or control vector
B	Body-fixed frame
I	Inertial frame

Operands and Math Symbols

$\times$	Cross product
$\vec{(\cdot)}$	Geometrical vector
$\hat{(\cdot)}$	Skew-symmetric matrix
$\lvert \cdot \rvert$	$\lVert \cdot \rVert_1$ norm
$\lVert \cdot \rVert$	Euclidean or $\lVert \cdot \rVert_2$ norm
T	Transpose of a vector or a matrix
$\frac{d(\circ)}{dt}\rvert_I$	Time derivative of a vector with respect to the inertial frame
$\frac{d(\circ)}{dt}\rvert_B$	Time derivative of a vector with respect to the body-fixed frame
$\angle$	Angle
$\dagger$	Complex conjugate
$\mathrm{diag}(\cdot)$	Components of a diagonal matrix
$\det(\cdot)$	Determinant
$\mathrm{rank}(\cdot)$	Rank of a matrix
I_n	$n \times n$ identity matrix
$0_{m \times n}$	$m \times n$ matrix with zero entries