TEXTS AND READINGS IN MATHEMATICS 6

Basic Ergodic Theory

Third Edition

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Preface

This book treats mainly some basic topics of ergodic theory in a revised form, bringing into focus its interactions with classical descriptive set theory more than is normally the practice. The presentation has a slow pace and can be read by anyone with a background in measure theory and point set topology. In particular, the first two chapters, the core of ergodic theory, can form a course of four to six lectures at third year B.Sc., M.Sc., or M.Phil. level in Indian Universities. I have borrowed freely from existing texts (with acknowledgements) but the overall theme of the book falls in the complement of these.

G. W. Mackey has emphasised the need to look at group actions also from a purely descriptive standpoint. This helps clarify ideas and leads to sharper theorems even for the case of a single transformation. With this in view, basic topics of ergodic theory such as the Poincaré recurrence lemma, induced automorphisms and Kakutani towers, compressibility and Hopf's theorem, the Ambrose representation of flows etc. are treated at the descriptive level before appearing in their measure theoretic or topological versions. In addition, topics centering around the Glimm-Effros theorem are discussed. These topics have so far not found a place in texts on ergodic theory. Dye's theorem, proved at the measure theoretic level in Chapter 11, when combined with some descriptive results of earlier chapters, becomes a very neat theorem of descriptive set theory.

A more advanced treatment of these topics is so far available only in the form of unpublished "Lectures on Definable Group Actions and Equivalence Relations", by A. Kechris (California Institute of Technology, Pasadena).

Professor Henry Helson has kindly edited the entire manuscript and suggested a number of corrections, greatly improving the language and the exposition. I am deeply indebted to him for this and many other acts of encouragement over the past several years.

It is a pleasure to acknowledge the consideration shown and help given by Dr. Mehroo Bengalee. She made the sabbatical leave available for this project during her tenure as the Vice Chancellor of University of Bombay. Finally, my sincere thanks go to V.Nandagopal for making his expertise with computers available in the preparation of this book.

M. G. Nadkarni

Preface to the Second Edition

In this edition a section on rank one automorphisms has been added to Chapter 7 and a brief discussion on the ergodic theorem due to Wiener and Wintner appears in Chapter 2. Typographical and other errors that were noticed or were brought to my notice have been corrected and the language has been changed in some places. The unpublished lectures of A. Kechris mentioned in the preface to the first edition have since appeared as "The Descriptive Set Theory of Polish Group Actions", H. Baker and A. Kechris, London Math. Soc. Lecture Note Series, 232, Cambridge University Press.

M. G. Nadkarni

Preface to the Third Edition

In this edition a chapter entitled 'Additional Topics' has been added. It gives Liouville's Theorem on the existence of invariant measure, entropy theory leading up to the Kolmogorov-Sinai Theorem, and the topological dynamics proof of van der Waerden's theorem on arithmetical progressions. It is a pleasure to acknowledge the help given by B. V. Rao and Joseph Mathew in this. These new topics are within the reach of interested undergraduates and beginning graduate students. Ankush Goswami pointed out some mathematical and typographical errors in the earlier edition. These and some other errors which were noticed have been corrected. I hope the new edition will be found useful.

I am much indebted to D. K. Jain of Hindustan Book Agency for suggesting a new edition of this book, and for monitoring its progress through timely emails and encouraging telephone calls. My sincere thanks also go to Vijesh Antony for quickly resolving my difficulties with the computer whenever I sought his help. Finally, thanks are due to Indian Institute of Technology, Indore, for visiting appointments during which this edition was prepared.

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