Natural Deduction, Hybrid Systems and Modal Logics

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Introduction

A good title should be informative enough to illuminate a potential reader on the content of a book. We hope that the present title gives at least some hints of what this book is about. The notion of natural deduction or modal logic are rather well known, but the notion of "hybrid system" certainly needs some explanation.

In short, this study may be seen as a kind of search for good deductive systems. We think of systems good in practice which may be applied with ease not only by well trained logicians but also, for example, by philosophers who need handy deductive tools accompanying their analyses. In particular, we are interested in providing systems that may be widely applied in teaching logic. Nowadays one may observe that several courses in "critical thinking" tend to eliminate courses in practical logic. On the other hand, logic is often taught as a strictly mathematical discipline in very demanding courses. It is important to fill the gap between these extrema, and the crucial ingredient of any course which is supposed to teach how to use logic, is certainly a suitable deductive system.

Since we address this work to a wide audience interested in applications of logic, we were trying to make it self-contained and accessible to a reader with no hard training in logic. The assumed reader should have some background in logic (an elementary course covering classical propositional and first-order logic with basics of set theory is enough) but not necessarily in modal logic.

The search for a good deductive system is realized in stages. Standard natural deduction for classical and free logic is investigated in the first place as the proper candidate for this aim. Closer inspection shows that natural deduction in standard form has some limitations (which should) to be overcome. In search of better deductive tools we introduce some modified versions of natural deduction and the so called hybrid systems – combinations of natural deduction with other kinds of calculi. Next, applications of natural deduction in standard and extended (hybrid) form to several modal logics are analyzed. Finally, a labelled approach is examined: first, in external form and then, in the strong internalised form, commonly called hybrid logics.

If the aim of the study is to find a good deductive system, then we should ask what does "good" mean with respect to a deductive system? This is a very general question, concerning a very vague notion that may cover many different things. We must underline again that we think of practically usable systems; theoretical considerations often lead to quite different desiderata. Let us try to make some preliminary list of particularly important properties.[1] A good practical deductive system should be:

- universal
- general
- extensive
- natural
- simple
- efficient

These terms are not technical (except the last perhaps) but informal and ambiguous. In the following explanations we stipulate meanings which we want to attribute to them, at least in this book.

We say that a proof system has *universal application* (or simply that it is universal) if it may be used to perform different deductive tasks. For example, a universal system allows not only constructing proofs but also showing that a formula is invalid by extracting a falsifying model. It makes possible to define proof search procedures, and even if the formalized logic is not decidable, it gives some ground for application in automated theorem proving. Typical tableau and resolution calculi, and to some extent, sequent calculi, satisfy this property, whereas axiomatic systems and natural deduction systems in their standard form, are not universal.

By *generality* of a system we mean the ability to apply different proof search strategies and to simulate in a direct fashion other kinds of systems.

[1] One may find other interesting lists of valuable properties e.g. in Avron [16], Wansing [280], Indrzejczak [143] or Poggiolesi [214]. But those lists are often more theoretically oriented and formulated mainly for several forms of sequent calculi, whereas the present one has a very general character and is more related to practice.

Several technical notions of simulation will be defined in Chapter 1. Informally, we mean that it is possible to apply deductive techniques from several sources in general systems. In consequence they may be used as a handy tool for a comparison of different proof-search strategies and their efficiency.

Extensiveness of a system is connected with the scope of its applicability. It means that the system provides a uniform deductive framework for the formalization of several nonclassical logics. Extensive systems yield handy tools for the investigation of different logics in a uniform fashion. So far, axiomatic systems are unquestionable winners in this category. But recent developments of sequent calculi, especially of nonstandard character (like display calculi or hypersequent calculi), or tableau calculi offer some hope in this respect.

A proof system is *natural* if its rules are modeled after traditional methods of inference, known from antiquity and used by humans in their common thinking, as well as in informal mathematical proofs. Natural deduction systems seem to satisfy this requirement better than other systems, because the latter are often limited to the use of special types of rules only, regulated rather by theoretical than practical needs. It is not surprising; Jaśkowski and Gentzen had just this goal in mind when they have constructed the first systems of this sort. Most variants and modifications introduced later were also generally connected with this idea.

Naturalness seems to be in close connection with *simplicity* of the system but this property is an example of particularly vague notion. Moreover, several possible senses are hardly subject to any objective criteria. Anyway, it is worth exploring. In the case of proof systems simplicity means, among other things:

1. simplicity of inference rules;
2. simplicity of the construction, and the limited number of elements of the whole system (easy to describe, to implement);
3. easy to follow proofs, readable for humans;
4. ability to construct short and direct proofs;
5. applicability of simple proof search strategies.

It is easy to observe that these features are rather independent and, moreover, sometimes they even tend to be in conflict. For instance, the possibility of building short and direct proofs is usually the result of the rich structure of the system. On the other hand, systems simple in the sense 1 or 2 are often unable to produce short and easy-to-follow (and to find) proofs. For example, axiom systems are certainly simple in the 1st and 2nd sense, which is the source of their success in metalogic. Axiomatic proofs also have, in a sense, a very simple structure, but it does not mean that they are readable or short, or easy to find! Natural deduction systems are usually simple in the sense 1, 3, and 5, but the price for that is a complex structure of the calculus. Similar remarks may be applied to other types of proof systems which will be discussed below.

The notion of *efficiency* is usually applied on the field of automated theorem proving and measured in terms of speed of running program or memory required for computation. But proofs generated by efficient programs may be quite long and complicated, while it is possible to find short and direct proofs with the help of some ingenuity. On the other hand, algorithms devised for finding short and readable proofs tend to be rather more complicated. It means, in terms of implementation, that suitable programs may require much more time and memory. We are concerned in this study with systems of practical utility designed for humans not for machines, so efficiency is a good thing, but not at the expense of other features like naturalness or simplicity. Such an approach does not exclude automatization but introduces considerable complications because a proof generated by a program should be readable for men.

Natural Deduction

In general it is not reasonable to claim that some type of a system is better than other ones. The best we can do is to evaluate systems as better than others with respect to some of the properties. For example, if we compare resolution and axiom systems, then certainly, the former is more efficient but the latter is more extensive. In fact, not all discussed properties may be used as serious criteria of evaluation. For example, when we compare different systems with respect to naturalness and simplicity (as explained above) such an evaluation must be subjective. But still it is reasonable to search for systems that may be assessed as having sufficiently high rating in all categories.

In our opinion, natural deduction systems (shortly called ND systems) seem to be the most promising but their abilities were not fully recognized so far. In this book we will try to justify our belief and to show that ND may be extended and generalized in many ways.

What supports our conviction is, in the first place, the richness of deductive apparatus of ND. It makes even standard ND quite general type of a system but, as we will show, simple modifications may considerably increase their generality. We have already mentioned in what sense ND systems may be called natural and simple. In fact, many existing ND systems may provoke an opinion that they are natural only by definition. We will focus on the systems which, in our opinion, are the simplest, the most natural, and moreover, they may be modified in several ways.

In common opinion ND systems are not very useful as a tool for proof search or for automation. It is a consequence of unnecessary limitation of standard ND which are defined as systems devised for proof construction only. In this form they are not universal since realization of other deductive tasks, like falsification of a formula, is not possible. If it is an essential property of ND, it would be more proper to call them "natural *proof* systems" instead of "natural *deduction*".[2] Fortunately, it is not difficult to make ND systems more universal.

The possibility of using ND systems as, e.g. working decision methods, opens the gate for the question of efficiency of ND and prospects for automation. In fact, ND systems are rather not considered as good candidates for that purpose. Rich deductive toolkit mentioned above may be rather troublesome for implementation. So it is not surprising that there are not so many provers based on ND (cf. Chapter 4). The unquestioned leader in the field is the family of resolution calculi, but programs based on resolution usually do not satisfy the requirement of readability of the output. Clearly, if the result (not the way leading to it) was the only important factor, it is inessential. But we stated above that for us rather the way than the result is more important, and it is also somewhat connected with automated deduction. Since 80s, more attention is paid to the construction of several forms of interactive programs for teaching logic, with some support built in. One may note that many programs of this sort, like MacLogic, Heterogenous Logic or Mizar, are based on ND systems.

These brief remarks on some desiderata concerning good proof systems are intended as an explanation of the leading role of ND systems in this book. Details that substantiate our claims will be found in the text. We did not touch so far the question of extensiveness of ND.

[2]By the way, it is more proper to use a term "automated deduction" instead of often applied "automated theorem proving" since modern programs are not bound to proving but realize a variety of deductive tasks, e.g. model checking.

Modal Logic

In this book we focus only on one class of nonclassical logic, however, very important one. The choice of *modal logics* as the field of application of investigated deductive techniques follows from the personal conviction that it is one of the most natural and useful class of logics. This view corresponds well with our program of searching for the most natural, practical and simple system of deduction. Let us notice however, that although we do not extend our results to other nonclassical logics, we take the term "modal logics" in a wider sense than it is usually applied. In particular, we consider:

1. Not only monomodal logics but also multimodal ones, in particular, bimodal temporal logics.

2. Apart from normal modal logics, also weaker classes of regular, monotonic and congruent logics.

3. Several versions of first-order nonmodal and modal logics.

4. Not only modal logics formulated in standard languages but hybrid modal logics in extended languages.

We believe that systems providing uniform characterization of this vast and diversified class of logics are extensive enough. Moreover, many results may be easily adapted to other nonclassical logics. In case of intuitionistic or some superintuitionistic logics it is straightforward, e.g. systems for modal logics of linear frames may be easily redefined to obtain a formalization of Dummett logic, in other cases it may need some work. But existing ND systems for many nonclassical logics not discussed in this book, e.g. for relevant logics in [5], may be seen as an additional evidence for our claim.

Hybrid Systems

Finally, we should explain what is meant by a *hybrid system*. Readers acquainted with modern modal logic may suspect that we mean deductive systems for *hybrid logics*. In fact, hybrid logics and deductive systems for them (including hybrid systems as well) will be also dealt with in this book. But the term hybrid system is by no means reserved for hybrid logics. Basically, in this book, this qualification is applied to several kinds of deductive systems developed by combination of elements taken from different sources. The construction of such systems is undertaken for optimalization of deduction.

The need for hybrid systems is closely connected with the evolution of logic. Development of computer sciences, investigations on artificial intelligence, problems with knowledge representation and management, and many other related factors, resulted in substantial changes in logic. Problems traditionally seen as technical and disregarded by logic community, currently provide the main areas of research. These changes, making logic less theoretical and more practically oriented, have strong impact on the methodology of deduction as well.

One of the cornerstones of modern formal logic is the distinction between syntactical and semantical investigations. What is defined in terms of the shape of expressions belongs to syntactical studies, what is defined in terms of interpretation belongs to semantics. A privileged position of *completeness* and *soundness* proofs in many studies is a very good witness of the importance of this distinction. In traditional reflection on methods of logic both approaches are treated as complementary. Syntactical methods are seen as tools for proving, whereas semantical methods are seen as tools of falsification. The introduction of systems of universal character, like tableaux, did not change this popular view. Even today in many modern textbooks one may find this tendency still alive.

Likewise, in traditional metalogic also *decidability* was taken seriously but practical aspects were rather ignored. It was enough to show that there is an algorithm which in finite time may always find an answer. Development of computer industry and growing interests in fast-running programs have changed the perspective. Even in case of undecidable logics one may construct reasonably efficient programs, thus undecidability does not exclude automation. On the other hand, even decidable theories may be practically nontractable if they require too much time or memory. A dynamic development of *complexity theory* is one of the signs of this trend. Searching for efficient methods seems to be much more important nowadays than proving completeness or decidability. From the standpoint of practical applications of logic (like, e.g. in *expert systems* or *computer-aided decision making*) one is rarely interested in complete systems but always in fast ones. The great success of *Horn clauses* may serve as an example of wide applicability of such "partial" logics.

Hence, in search of optimal logical tools, problems of theoretical purity are not so important. Good results are often obtained by free combination of tools taken from several sources. Deductive systems resulting from such operations are called here hybrid systems. One may distinguish at least two types of hybridization:

1. An introduction of syntactically encoded elements of semantics into the realm of deductive (syntactic) system.

2. A combination of different types of deductive systems which were originally devised for the realization of different deductive tasks.

Both types of hybridization are of unequal status. Using elements of semantics in deductive systems has rather long tradition. In fact, the application of semantical information was always a natural way of supporting deduction. For example, it often appeared in several forms of diagrams representing (partial) interpretation.[3] Development of mathematical logic, and, in particular, the popularity of Hilbert program in its first phase, led to absolutization of the distinction between syntax and semantics, as we have already remarked. But this is not only artificial; it may be harmful, both from the standpoint of optimal tools of deduction, and of the practice of teaching logic. As for the latter case one may note a great popularity of such programs for teaching logic like *Tarski's World* or *Heterogenous Logic* (both due to Barwise and Etchemendy). Generally, it seems that several attempts at erasing the border between semantics and syntax appeared already in the first half of XX century. One may compare in this respect the approaches of Beth or Quine in their books on logical methods [27, 226].

Hybridization of this type is realized in different ways in modern logic. Some of the techniques applied in systems for modal logics, like introduction of diagrams, rectangles, brackets into tableaux, or generalizations of sequents (hypersequents, multisequents, e.t.c.) will be shortly described in Chapters 7 and 8. But the main technique for syntactical encoding of semantics considered in the book is labelling. It is superior to other techniques in two respects. First, labels may take different forms dependant on the kind of (elements of) semantics we want to use and on the grade of semantical involvement. Second, labels may be applied in combination with any kind of deductive system. In particular, deductive systems for hybrid logics may be also seen as an important example of labelled systems of a very special sort.

The second type of hybridization was not widely applied so far but it may lead to the great improvement of traditionally recognized deductive

[3] One may find a survey of such techniques in, e.g. Bocheński [43], Kneale [163], Marciszewski and Murawski [181].

systems. We have already remarked that no single type of a system may be viewed as better than others in all respects. But a reasonable combination of rules, techniques, strategies taken from different sources, may result in a system which inherits good properties of all parents. Such systems may realize different deductive tasks in satisfying way without the necessity of changing the basic system. The attempts of this kind are undertaken for the creation of integrated environments for working with different logics or theories. We mean here a variety of programs qualified as a *logical frame* or *generic prover*, like Isabelle, Automath, Otter or Mizar.[4] In our opinion, ND systems, due to their rich assortment of deductive tools, are best prepared to work as an uniform basis for the integration of other types of rules and techniques. In what follows we will provide some examples of such ND-based hybrid systems.

Overview

The book may be divided into three parts. The first part comprises 4 chapters and lays down the foundational issues concerning ND systems in standard and extended form. Chapter 1 introduces classical and free logics and several technical notions concerning deductive systems, rules e.t.c. The next Chapter presents a short history and a systematization of several forms of standard ND systems. The distinction between different formats of ND is essential since, as we shall see, not all of them may be used as a suitable basis for extension to modal or other nonclassical logics. In this book we focus on ND in Jaśkowski's format, in the version called KM due to Kalish and Montague. Several variants and generalizations of KM are provided in due course, but it is usually pointed out whether these modifications may be adjusted also to other versions of ND. Quite independently of the chosen ND format a lot of different formalizations of first-order classical and free logic were proposed. This is in contrast to rather uniform treatment provided for sequent calculi or tableau systems. We characterize the most important of these several solutions either. The next step is to overcome limitations of standard ND. For that reason in Chapter 3 we survey different types of deductive systems that may provide inspiration for generalizations of ND. It is a subject of Chapter 4, introducing analytic and universal versions of standard ND able to simulate tableau systems and KE system of D'Agostino and Mondadori. Finally, we introduce RND (resolution based ND) oper-

[4]Cf. Basin [23] for a good introduction into this theme.

ating on clauses, a powerful generalization of ND directly simulating many clausal systems like resolution calculi or Davis/Putnam method.

The second part (Chapters 5, 6, and 7) is concerned with modal logics and their formalization in standard ND systems. Chapter 5 provides a short introduction to the families of modal logics which are dealt with in this book. In Chapter 6 we present standard formalization of modal logics in sequent and tableau calculi and survey different approaches to construction of ND for modal logics. One of them, due to Fitch, is then exploited because it is the most extensive. We provide a few versions of KM system based on Fitch's approach. Chapter 7 shows some other possible extensions based on standard approach; in particular, two variants of RND for modal logics are introduced. Some theoretical problems, as well as limitations of standard method, are also discussed. We end up with the introduction of some nonstandard approaches to formalization of modal logics.

The third part is devoted to detailed exploration of one of the nonstandard approaches to formalization of modal logics, based on the application of labels. The term labelling is treated here in the very wide sense, comprising also hybrid logics, seen as a form of internal labelling. In Chapter 8 we start with general remarks on several forms of labelled systems and focus on the popular solution due to Fitting. In particular, we demonstrate that it is possible to apply this technique also to weak modal logics and combine labels not only with ND but also with RND. Chapter 9 shows that Fitting's labels may be also used to formalization of logics characterized by linear frames. This group is treated separately not only because of its importance (for instance in formalization of linear time) but because of specific technical problems we encounter. Moreover, the solution we propose may be also applied to other important logics determined by frames described by the so called universal implications. Chapter 10 has more technical character. We present a series of constructive completeness proofs for many analytic labelled ND systems, based on suitable proof search procedures. Some questions of efficiency and optimization are also briefly discussed.

Although systems based on the application of Fitting's labels are more extensive than standard ND systems, they still suffer from some limitations. The use of stronger forms of labelling leads to more extensive solutions. Particularly interesting form is provided by the use of hybrid logics. Chapter 11 is a short introduction to varieties of languages covered by this term. In Chapter 12 we survey deductive systems provided for hybrid logics and provide ND and RND systems of a very extensive character. In fact, these two chapters may be treated as a separate part, where the most extensive results are eventually offered.

Thus one may note that we try to extend the possibilities of ND systems gradually. Standard ND is first modified to obtain universal and analytic form for classical logic. Next we examine different ways of extending standard ND to modal logics. Then we add external labels to improve extensivity and to obtain an analytic ND for modal logics. At the end we introduce stronger (internalised) form of labelling which is the most extensive solution.

Finally, we should say a few words on what this book is not about.

Although there are parts of it where some algorithms are stated it is not a book on automated natural deduction. ND is treated here as a practical deductive tool of a great pedagogical value, useful rather for pen and paper implementation. Decision procedures for some systems are defined for the need of constructive completeness proofs rather than for real application. We are interested in proof search but performed by means of natural and simple rules. Hence although we show that resolution may be simulated in ND, no discussion of unification, and skolemization is provided.[5] In our opinion these techniques, despite their efficiency, are not natural; their application may speed up a proof, but not make it more readable. We also do not describe resolution proof search strategies since they are presented in many places; the interested reader is encouraged to test for himself how they work in the setting of RND.

Again, for the same reasons we do not take up theoretical questions connected with ND. In particular, problems of normalization of proofs are sometimes signalled but no systematic treatment is provided. In our opinion these results are very important but have rather theoretical character, whereas this study is concerned with these aspects of ND which may simplify doing proofs by hand. Also, no discussion of matters concerning encoding ND in lambda calculus through Curry-Howard isomorphism, e.t.c. is provided. These are certainly very important aspects of investigation on ND but again of theoretical character and very technical in nature. Inclusion of such considerations would result in another book, certainly harder to write. Instead, the question of analyticity is investigated as having a serious impact on the practice of proof search.

In the search of universal, general and extensive variants of ND we make free use of other types of deductive systems. But this is not a book on other

[5] One may find a presentation of respective forms of these techniques suitable for ND in Pollock [217].

proof systems so many of them are not even mentioned. In particular, our presentation of deductive systems for modal logics is far from being complete. Only these systems are introduced that satisfy at least one of the following criteria:

- They may be combined in some way with ND systems.
- They are extensive (at least potentially they have wide scope of application).
- They offer a possibility of formalization of bimodal temporal logics.
- They provide rules for logics of linear models.

The first criterion is obvious since we have chosen ND systems as a basis for obtaining uniform, general and universal hybrid systems. Remaining ones are connected with the application to modal logics.

Since the book is intended to a wide audience interested in practical application of logical tools, a detailed statement of technicalities is often avoided. In particular, if some definitions or proofs go along similar lines as those previously stated, the details are usually omitted and we rather encourage the reader to do it as an exercise. Certainly, the readers interested only in the application of systems, not in proving their properties, may skip respective parts of the text. On the other hand, there are some harder and more demanding parts which may be of interest for more technically oriented reader. They are generally connected with establishing adequacy of described ND systems. Since we are concerned with practical application of ND we were trying to reduce to minimum such metalogical proofs. Completeness results in most cases are obtained by simulation of other complete deductive systems, stated in quite an informal way and rather easy to follow. But in some cases there is no possibility to simulate something better known and full completeness proof must be stated instead (e.g. for labelled ND for linear logics in Chapter 9). Similarly, proofs of decidability based on proof search procedures defined for analytic version of ND in Chapters 4 and 10 are stated in detail, at least in these parts where specific features of ND play the crucial role. We have paid an attention also to soundness proofs for ND systems. Obviously, the reader interested only in finding working ND systems may skip these parts of the text without a loss.

This book is based on the author's habilitation published in Polish in 2006, and defended succesfully in 2007. Almost half of the present text is a (rather free and revised) translation of much of it. But this work contains substantial enlargement of that book. In particular, we have added here a treatment of first-order (classical, free, modal) logics, formalizations of weak (congruent, monotonic, regular) modal logics, and extended strongly the exposition of standard ND for modal logics and a treatment of hybrid logics. As a result, the book contains some parts based not on the habilitation but rather on some of my other papers devoted to natural deduction and modal logics. All dependencies on my other papers are credited in the text; in particular, Chapters 11 and 12 are heavily based on [155].

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