Part II

Examples of Learning Through Teaching: Pedagogical Mathematics

Interlude 1

Part I of this book provided a theoretical background on teachers’ learning through teaching (LTT). However, it was not devoid of examples. In fact, the chapters by Leikin, Mason, and Leikin and Zazkis, despite their focus on theoretical or methodological issues, provided numerous examples of teachers’ learning. Similarly, the authors of chapters in Parts II and III, despite their focus on particular instances of LTT, provide further theoretical and methodological considerations. They employ a variety of theoretical perspectives as a lens for describing and analyzing examples of learning through teaching.

As mentioned in the introduction to this volume, we found that distinguishing between the mathematics and pedagogy that have been learned by a teacher through teaching was extremely complex. We decided to address this complexity by using the notions of mathematical pedagogy and pedagogical mathematics as introduced by Mason (2007). In the chapter that opens Part II of this volume, Zazkis explicitly analyses the interrelationship between mathematics and pedagogy as used by the teacher educator with the purpose of developing the mathematics and pedagogy of prospective mathematics teachers. Zazkis demonstrates how the examination of this interrelationship led to mathematical discoveries and didactical insights. The theoretical framework employed in this chapter exemplifies the complexity of distinctions between mathematics and pedagogy that teachers learn and endorses the use of Mason’s constructs – mathematical pedagogy and pedagogical mathematics – in structuring our book.

The chapters in Part II focus on pedagogical mathematics; they describe particular cases of LTT and analyze the ways in which learning occurs.

Pedagogical Mathematics

Let us consider the particular examples presented by the authors.

For several teachers featured in these chapters, learning included solving a new (for them) mathematical problem, or learning a new solution to a known problem,
while connecting several mathematical ideas. Several examples of this kind are presented in Part I of this volume. Rachel, the teacher in Leikin’s chapter (in Part I), learned a number of new solutions for a given problem in geometry. Consequently, she extended her solution space – a notion introduced by Leikin to explain some mechanisms of LTT and is exemplified by Rachel’s case. Moreover, Rachel continued her mathematical explorations, based on her lesson with prospective teachers, and discovered new for herself mathematical facts. Shelly and Einat, the teachers in Leikin and Zazkis’ chapter, connected calculus with mathematical induction, and a conic section with maximum-minimum problems, respectively. In this same chapter, based on a student’s inquiry, Eva extended a given “familiar” theorem to include a special case of an isosceles triangle.

Rina (in Zazkis’ chapter) learned a new theorem related to invariances in affine transformations. Michael, the teacher in Kieran and Guzman’s study, acquired a new student-generated solution to the task of proving that \((x+1)^n-1\) is always a factor of \(x^n-1\). Students’ proofs also expanded Michael’s solution space. Marcelo and Rubia (in Borba and Zulatto’s chapter) developed a new explanation that distinguishes the “look-alike” conic sections of parabola and half-hyperbola. They also designed a technology-based illustration of their explanation. Ms. Alley and Ms. Lewis (in Markus and Chazan’s study) enhanced their own knowledge of equations in two variables as they explored this mathematical content for teaching.

Jackiw and Sinclair provide a very unique perspective on learning mathematics. In their study, learning mathematics involves learning mathematical discourse, where the computer plays the role of the traditional student and the teacher’s role is given to the hypothetical user-learner of the computer software, which can be either a teacher or a student.

In all these examples of enhanced mathematical knowledge, learning mathematics followed critical pedagogical events: a repeated mistake of students (Zazkis), an unexpected feedback from software (Jackiw & Sinclair), learners’ questions or suggestions (Leikin & Zazkis, Leikin, and Borba & Zulatto), and acknowledgement of students’ difficulty and a search for ways to help students build their mathematical understanding of the algebra (Markus & Chazan). Implementation of technological tools that allowed learners to engage in mathematical explorations (Kieran & Guzman and Borba & Zulatto) seems to intensify teachers’ learning of mathematics. Moreover, we see in researchers’ reports (e.g., Kieran & Guzman and Marcus & Chazan) and self-reports (e.g., Zazkis and Borba & Zulatto) that teachers’ learning not only followed but also resulted in new pedagogical approaches, activities, or explanations. That is to say, the newly-learned mathematical content became a part of these teachers’ pedagogical repertoire.

By considering the examples mentioned above as examples of pedagogical mathematics, we are extending Mason’s definition. (Recall: “Pedagogical mathematics involves mathematical explorations useful for, and arising from, pedagogical considerations”). These examples indeed arise from pedagogical considerations, even when these considerations are not mentioned explicitly. They are triggered by interactions with students, the desire to accommodate students’ ideas and queries, as well as the flexibility in doing so. However, we see “explorations” as only one
of several indicators of pedagogical mathematics. Other indicators include mathematical problem posing, enriching solution spaces of a mathematical problem, extending the repertoire of explanations, reinforcing mathematical discourse, and fostering mathematical connections. These examples also demonstrate strength in teachers’ prior mathematical knowledge. This strength is essential in developing and accommodating new ideas and, as such, is essential in developing pedagogical mathematics.

To summarize, in our view, pedagogical mathematics involves a broad range of mathematical objects, actions, activities, and tools constructed in a pedagogical context and useful in teaching. The chapters in Part II instantiate teachers’ learning of mathematics through their own teaching, where all notions – teachers, mathematics, and teaching – are interpreted broadly.

References