Nonlinear Least Squares for Inverse Problems

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Nonlinear Least Squares for Inverse Problems

Theoretical Foundations and Step-by-Step Guide for Applications

With 25 Figures

Springer

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To my wife Annette

Preface

The domain of inverse problems has experienced a rapid expansion, driven by the increase in computing power and the progress in numerical modeling. When I started working on this domain years ago, I became somehow frustrated to see that my friends working on modeling where producing existence, uniqueness, and stability results for the solution of their equations, but that I was most of the time limited, because of the nonlinearity of the problem, to prove that my least squares objective function was differentiable.... But with my experience growing, I became convinced that, after the inverse problem has been properly trimmed, the *final least squares problem*, the one solved on the computer, should be *Quadratically (Q)-wellposed*, that is, both *wellposed* and *optimizable*: optimizability ensures that a global minimizer of the least squares function can actually be found using efficient local optimization algorithms, and wellposedness that this minimizer is stable with respect to perturbation of the data.

But the vast majority of inverse problems are nonlinear, and the classical mathematical tools available for their analysis fail to bring answers to these crucial questions: for example, compactness will ensure existence, but provides no uniqueness results, and brings no information on the presence or absence of parasitic local minima or stationary points ....

This book is partly a consequence of this early frustration: a first objective is to present a *geometrical theory* for the analysis of NLS problems from the point of view of *Q-wellposedness*: for an attainable set with finite curvature, this theory provides an estimation of the size of the admissible parameter set and of the error level on the data for which Q-wellposedness holds. The various regularization techniques used to trim the inverse problem can then be checked against their ability to produce the desirable Q-wellposed problems.

The second objective of the book is to give a detailed presentation of important practical issues for the resolution of NLS problems: sensitivity functions and adjoint state methods for the computations of derivatives, choice of optimization parameters (calibration, sensitivity analysis, multiscale and/or adaptive parameterization), organization of the inversion code, and choice of the descent step for the minimization algorithm. Most of this material is seldom presented in detail, because it is quite elementary from the mathematical point of view, and has usually to be rediscovered by trial-and-error!

As one can see from these objectives, this book does not pretend to give an exhaustive panorama of nonlinear inverse problems, but merely to present the author's view and experience on the subject. Alternative approaches, when known, are mentioned and referenced, but not developed. The book is organized in two parts, which can be read independently:

Part I (Chaps. 1–5) is devoted to the step-by-step resolution and analysis of NLS inverse problems. It should be of interest to scientists of various application fields interested in the practical resolution of inverse problems, as well as to applied mathematicians interested also in their analysis. The required background is a good knowledge of Hilbert spaces, and some notions of functional analysis if one is interested in the infinite dimensional examples. The elements of the geometrical theory of Part II, which are necessary for the Q-wellposedness analysis, are presented without demonstration, but in an as-intuitive-as-possible way, at the beginning of Chap. 4, so that it is not necessary to read Part II, which is quite technical.

Part II (Chaps. 6–8) presents the geometric theory of quasi-convex and strictly quasi-convex sets, which are the basis for the results of Chaps. 4 and 5. These sets possess a neighborhood where the projection is well-behaved, and can be recognized by their finite curvature and limited deflection. This part should be of interest to those more interested in the theory of projection on nonconvex sets. It requires familiarity with Hilbert spaces and functional analysis. The material of Part II was scattered in various papers with different notations. It is presented for the first time in this book in a progressive and coherent approach, which benefits from substantial enhancements and simplifications in the definition of strictly quasi-convex sets.

To facilitate a top-to-bottom approach of the subject, each chapter starts with an overview of the concepts and results developed herein – at the price of some repetition between the overview and the main corpus of the chapter.... Also, we have tried to make the index more user-friendly, all indexed words or expressions are emphasized in the text (but not all emphasized words are indexed!).

I express my thanks to my colleagues, and in particular to François Clement, Karl Kunisch, and Hend Benameur for the stimulating discussions we had over all these years, and for the pleasure I found interacting with them.

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