

# Mathematical Approaches to Biological Systems



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Editors

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Networks, Oscillations, and Collective  
Motions



Springer

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# Preface

Mathematical approaches to biological systems are now fast-growing research endeavors. Although mathematics has traditionally found more applications to physics, many of the same techniques and concepts developed are increasing our abilities for exploring various aspects of biological systems.

The aim of this book is to present some aspects of these mathematical approaches under the keywords of “networks”, “oscillations”, and “collective behaviors”. Although each of these subjects is often discussed in separate texts, we hope that readers can sense their mutual connections in concepts.

Another feature of this collection is its connection to traditional Japanese research in the related areas. All of the seven chapters carry the tradition of Japanese research of mathematics, engineering, and physics, educated and/or influenced by schools of such pioneering researchers as E. Teramoto, R. Kubo, Y. Kuramoto, S. Amari, K. Kaneko, and S. Oishi. Readers can enjoy the way that unique cultural tones and approaches in research have been passed down.

Robert Brown was a botanist interested in the movements of particles observed using a microscope. His original intention was to find basic constituents of biological systems. Although Brown concluded that what he observed were not “biological particles”, his study of what is now referred to as Brownian motion has had a huge impact in mathematics, physics, and chemistry. For example, Brownian motion is the cornerstone that solidified the concept of atoms and was used by J.B. Perrin to estimate Avogadro numbers through the theory of A. Einstein. It is interesting to note that this foundation of physics and chemistry was initiated by biological research. The collection of papers in this book builds on the theme that mathematical approaches to biological systems are not one-way applications of mathematics to biology. It will be our pleasure if readers of these collected works are inspired to feel the potential of developments into various fields. Let us start by introducing each contribution to this book.

Many biological and physiological feedback systems involve time delay and a threshold of sensing (dead zone). Together, these properties lead to switching-type time-delayed feedback controllers which can produce rather unexpected results. Milton, who worked under Prof. E. Teramoto in the 1970s, together with Insperger and Stepan investigate this topic for different systems including bobwhite quail populations in northern Wisconsin, digitally controlled machinery with micro-chaos, and feedback control of human balance. The most notable finding of the mathematical models is the fact that transient time-dependent bounded solutions (transient stabilization) can arise even for parameter ranges where the equilibrium is asymptotically unstable. In the context of balance control, this means that the combination of a sensory threshold with a time-delayed sampled feedback can increase the range of parameter values for which balance can be maintained, at least transiently. This may lead to a new class of control mechanisms that are suited for control in the presence of delay and noise.

Networks and interactions among basic units, such as neurons, cells, and proteins, are important in biological systems. However, in order for networks to produce useful functions, it is necessary that the network interactions be robust in the face of external perturbations. Thus an understanding of the robustness of biological networks from the point of view of dynamical systems is an important and challenging topic. Tanaka, Morino, and Aihara address this issue in their chapter. First, they discuss structural and dynamical robustness of networks and map out a framework for studying their dynamical robustness. Then, concrete examples from coupled oscillator networks, and neural network models are investigated thoroughly over different types of network structures. Neurons with a high firing rate are found to be important. The efficacy of their investigation scheme for understanding biological network systems is illustrated by the demonstration that dynamical robustness can arise by altering neural firing rates through synaptic couplings.

Modeling of neurons and neural networks using a variety of theoretical approaches has a long history. For example, numerous neural computer algorithms and software applications, such as pattern recognition and learning, have been developed. However, a device that computes using the computational rules of living nervous systems, i.e., a brain computer, has yet to be built. Matsubara and Torikai take an important step in this direction by using a hardware-oriented neuron modeling approach to develop an asynchronous sequential logic neuron model. Their model can successfully reproduce the nonlinear dynamics of biological neurons with its on-chip learning capabilities. A future vision toward the brain-like systems is also presented.

In many wireless communications, such as cell phones, it is quite important that the device can respond efficiently to incoming signals. Therefore it is important to construct design principles that achieve such efficient responses. In his chapter, Hisa-Aki Tanaka tackles this issue from the point of view of entrainment. Nonlinear oscillators exhibit injection locking to external periodic forcing. For a given

oscillator it is still an open problem to establish an ideally efficient injection locking. Tanaka presents a universal mechanism governing the entrainment limit under weak forcing. This demonstration provides a guide towards the design of efficient injection-locking methods under various situations and constraints. The theory is illustrated using a model for a Hodgkin–Huxley neuron.

Open complex systems, such as ecosystems and social communities, exhibit global properties. For example, there can be an emergence and persistence of complexity as new elements are introduced. Shimada asks under what conditions open complex systems can grow or remain in a stationary state. By investing a simple model system for an open complex system, he shows that the crucial parameter is the average number of interactions per unit. Using a mean field analysis it is possible to show that there is a moderate range of interactions where the complexities grow toward infinity. Outside of this parameter range, the system remains at a finite size. Shimada's work makes a contrast with the classical diversity–stability relation for dynamical systems and with other complex networks models. Yet, it captures some of the aspects of real complex systems, such as ecosystems viewed from lifetime distribution of fossil data.

In the work by Ogihara, Yamanaka, Akino, Izumi, Awazu, and Nishimori, the intriguing topic of communication of ants is investigated. One of the most popular and investigated topics is foraging. Various species of ants create long foraging trails between nests and food sources. However, some species do not form trails but exhibit tandem-running, namely, the ants follow a few guiding nest mates. Ants perform these behaviors based on sensing cues. In order to function in a temporally fluctuation environment with their limited sensing ability, some species of ants make a combined use of different cues, such as chemicals, and visual lights. By means of experiments and mathematical modeling of paths between a home and a food source, the authors show that ant foragers use a simple but systematic approach based on the switching between primarily reliance cues, which depends on the degree of conflict between different cues.

One of the common activities in animal behavior is hunting and evading. This problem of “chases and escapes” has been considered as a mathematical problem since the eighteenth century. Kamimura, Matsumoto, and Ohira begin their chapter by presenting a historical background of one-to-one chase and escape. Then a brief introduction of collective behavior of self-propelled particles, in particular, a theoretical traffic model called the optimal velocity model, is used to motivate their recent work on group chase and escape. The most notable feature of their model is that complex group patterns can emerge despite the lack of communication among peers. In particular, the preys form an aggregate in escaping, while the predators develop surrounding patterns of chasing. Parameters to capture such collective behaviors are also introduced and the effects of fluctuations are investigated. Extensions of their simple models are also discussed.

The collected works ranging from networks to collective behaviors all attempt to open new directions of research in the mathematics of biological systems. As editors, it has been a pleasure to have been in contact with these fine researchers, and we hope that readers will have the same feeling.

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