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Slow Rarefied Flows

Theory and Application to Micro-Electro-Mechanical Systems

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Preface

This volume is intended to cover the present status of the mathematical tools used to deal with problems related to slow rarefied flows. The meaning and usefulness of the subject, and the extent to which it is covered in the book, are discussed in some detail in the introduction. In short, I tried to present the basic concepts and the techniques used in probing mathematical questions and problems which arise when studying slow rarefied flows in environmental sciences and micromachines. For the book to be up-to-date without being excessively large, it was necessary to omit some topics, which are treated elsewhere, as indicated in the introduction and, whenever the need arises, in the various chapters of this volume. Their omission does not alter the aim of the book, to provide an understanding of the essential mathematical tools required to deal with slow rarefied flows and give the background for a study of the original literature.

Although I have tried to give a rather complete bibliographical coverage, the choice of the topics and of the references certainly reflects a personal bias and I apologize in advance for any omission.

I wish to thank Lorenzo Valdettaro, Antonella Abbà, Silva Lorenzani and Paolo Barbante for their help with pictures and especially Professor Ching Shen for his permission to reproduce his pictures on microchannel flows.

Milano, December 2005 Carlo Cercignani

Introduction

Rarefied gas dynamics can be defined as the study of gas flows in which the average value of the distance between two subsequent collisions of a molecule (the so-called mean free path) is not negligible in comparison with a length typical of the structure of the flow being considered, e.g., the thickness of a microchannel or the radius of curvature of the nose of a space shuttle. Thus it intrinsically requires the use of statistical ideas typical of the kinetic theory of gases as embodied in the integro-differential equation proposed by Boltzmann in 1872 and bearing his name.

Rarefied gas dynamics has existed, in principle, since the 19th century, but came in the foreground with space exploration. One can even give a birthdate, July 1958, when the first international symposium on rarefied gas dynamics was held in Nice (France). Since then, these symposia have been held regularly every second year. When glancing through the corresponding proceedings, one should not be surprised to find a shift of topics. The first few volumes contain a considerable amount of experimental papers and the theoretical papers contain very general surveys on the Boltzmann equation that rules the evolution of rarefied flows, but very few papers dealing with explicit solutions of some elementary problems. The first numerical solutions of some interest appear in 1962, but still in the late 1960s were few in number and not so accurate. Then one witnesses the reduction of experimental work and the increasing importance of numerical simulation. In the most recent volumes, experiments occupy just a few pages of the proceedings. This is compensated for by the fact that numerical simulations have spread through all the subfields, indicating the maturity reached by the theoretical understanding of the subject. Increasingly complicated phenomena, such as reacting flows or evaporation and condensation, are the object of widespread interest.

The mathematical theory of the Boltzmann equation goes back to such illustrious mathematicians as Hilbert and Carleman and is mentioned in the motivation of the Fields medal awarded to P.L. Lions in 1994. Some details of this theory will be presented in this book. The present introduction is mainly devoted to explain why this equation is so important for applications. We also remark that this book, although describing a well-defined topic, can serve two sets of readers: those more interested in the basic mathematical theory and those more interested in applications. The former might restrict themselves to Chapters 1–4, the latter to Chapter 1, the first section of Chapter 4 and Chapters 5–7.

In addition to space research, rarefied gas dynamics is also required in the area of environmental problems. Understanding and controlling the formation, motion, reactions and evolution of particles of varying composition and shapes, ranging from a diameter of the order of .001 μm to 50 μm, as well as their space-time distribution under gradients of concentration, pressure, temperature and the action of radiation, has grown in importance, because of the increasing awareness of the local and global problems related to the emission of particles from electric power plants, chemical plants, vehicles as well as of the role played by small particles in the formation of fog and clouds, in the release of radioactivity from nuclear reactor accidents, and in the problems arising from the exhaust streams of aerosol reactors, such as those used to produce optical fibers, catalysts, ceramics, silicon chips and carbon whiskers.

One cubic centimeter of atmospheric air at ground level contains approximately 2.5×10^{19} molecules. About a thousand of them may be charged (ions). A typical molecular diameter is 3×10^{-10} m (3×10^{-4} μm) and the average distance between the molecules is about ten times as much. The mean free path is of the order of 10^{-8} m, or 10^{-2} μm. In addition to molecules and ions, one cubic centimeter of air also contains a significant number of particles varying in size, as indicated above. In relatively clean air, the number of these particles can be 10^5 or more, including pollen, bacteria, dust, and industrial emissions. They can be both beneficial and detrimental, and arise from a number of natural sources as well as from the activities of all living organisms, especially humans. The particles can have complex chemical compositions and shapes, and may even be toxic or radioactive. A suspension of particles in a gas is known as an aerosol. Atmospheric aerosols are of global interest and have important impact on our lives. Aerosols are also of great interest in numerous scientific and engineering applications.

A third area of application of rarefied gas dynamics has emerged in the last few years and will be discussed in detail in the last chapter of the present book. Small size machines, called micromachines, are being designed and built. Their typical sizes range from a few microns to a few millimeters. Rarefied flow phenomena that are more or less laboratory curiosities in machines of more usual size can form the basis of important systems in the micromechanical domain. In fact, rarefied gas flows occur in many micro-electro-mechanical systems (MEMS), such as actuators, microturbines, gas chromatographs, and micro air vehicles (MAVs). A correct prediction of these flows is important to design and develop MEMS. Nanoscale design occurs for computer components as well and is no longer limited to chip technology but extends to mechanical devices as well. In a modern disk drive, the read/write head floats at distances of the order of 50 nm above the surface of the spinning platter. The prediction of the vertical force on the head (as obtained from the pressure distribution in the gas) is a crucial design calculation since the head will not accurately read or write if it flies too high. If the head flies too low, it can catastrophically collide against the platter. Micro-channels may have further computer applications because they are supposed to dissipate the heat generated in microchips more effectively than fans, and may be used as a more practical cooling system in integrated circuit chips.

Since, as these examples indicate, micro-devices are gaining popularity both in commercial applications and in scientific research, there exists a rapidly growing interest in improving the conventional design techniques related with these devices. Micro-devices are often operated in gaseous environments (typically air), and thus their performances are affected by the gas around them. The numerical simulation of all these flows cannot be performed with the Navier–Stokes equations (or the related Reynolds equation for a slider air bearing) because the smallest characteristic length of MEMS or of the thin air film occurring in a computer drive is comparable with (or smaller than) the mean free path of the gas molecules. For this reason the continuum equations are no longer valid and the Boltzmann equation must be invoked to understand and compute the rarefied flows related to these devices.

Numerical methods based on this equation are generally numerically expensive especially when the flow to be considered progresses from free molecular, through transitional, to continuum regions. Since these flows, contrary to the flow past space vehicles, are usually at low Mach number, the use of the linearized Boltzmann equation is permissible and this revives old methods developed in the sixties and seventies of the 20th century to deal with this equation.

Among the rarefied flows of interest, one should not forget the design and simulation of the aerosol reactors, used to produce optical fibers, catalysts, ceramics, silicon chips and carbon whiskers, which have been mentioned above as sources of air pollution. A further area of interest occurs in the vacuum industry. Although this area has existed for a long time, the expense of the early computations with kinetic theory precluded applications of numerical methods. The latter could develop only in the context of the aerospace industry, because the big budgets required till recently were available only there.

The present volume is an attempt to cover the mathematical results and techniques to deal with rarefied flows when the speeds are small with respect to the sound speed. The mathematical theory is much more advanced in this case and provides a rigorous justification for the use of the linearized Boltzmann equation, which avoids costly simulations based on Monte Carlo methods.

After introducing the Boltzmann equation in Chapter 1, we shall survey the rigorous theorems on validity and existence in Chapter 2. Chapter 3 is devoted to the basic existence theory for flows close to equilibria in an infinite expanse of gas or in a periodic box. Chapter 4 deals with more realistic boundary conditions and Chapter 5 deals with the techniques used to solve problems in the simple but extremely important case of a slab geometry. Chapter 6 discusses problems in three dimensions and Chapter 7 is devoted to the recent contributions to rarefied lubrication theory with particular attention to applications to MEMS.