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Operator Theory, Systems Theory and Scattering Theory: Multidimensional Generalizations

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Editorial Introduction

Daniel Alpay and Victor Vinnikov

La séduction de certains problèmes vient de leur défaut de rigueur, comme des opinions discordantes qu'ils suscitent: autant de difficultés dont s'entiche l'amateur d'Insoluble.

(Cioran, La tentation d'exister, [29, p. 230])

This volume contains a selection of papers on various aspects of operator theory in the multi-dimensional case. This last term includes a wide range of situations and we review the one variable case first.

An important player in the single variable theory is a contractive analytic function on the open unit disk. Such functions, often called Schur functions, have a rich theory of their own, especially in connection with the classical interpolation problems. They also have different facets arising from their appearance in different areas, in particular as:

- *characteristic operator functions*, in operator model theory. Pioneering works include the works of Livšic and his collaborators [54], [55], [25], of Sz. Nagy and Foiaş [61] and of de Branges and Rovnyak [23], [22].
- *scattering functions*, in scattering theory. We mention in particular the Lax–Phillips approach (see [53]), the approach of de Branges and Rovnyak (see [22]) and the inverse scattering problem of network theory [38]; for a solution of the latter using reproducing kernel Hilbert space methods, see [8], [9].
- *transfer functions*, in system theory. It follows from the Bochner–Chandrasekharan theorem that a system is linear, time-invariant, and dissipative if and only if it has a transfer function which is a Schur function. For more general systems (even multi-dimensional ones) one can make use of Schwartz' kernel theorem (see [76], [52]) to get the characterisation of invariance under translation; see [83, p. 89, p. 130].

There are many quite different approaches to the study of Schur functions, their various incarnations and related problems, yet it is basically true that there is only one underlying theory.

One natural extension of the single variable theory is the time varying case, where one (roughly speaking) replaces the complex numbers by diagonal operators and the complex variable by a shift operator; see [7], [39].

The time varying case is still essentially a one variable theory, and the various approaches of the standard one variable theory generalize together with their interrelations. On the other hand, in the multi-dimensional case there is no longer a single underlying theory, but rather different theories, some of them loosely connected and some not connected at all. In fact, depending on which facet of the one-dimensional case we want to generalize we are led to completely different objects and borderlines between the various theories are sometimes vague. The directions represented in this volume include:

- *Interpolation and realization theory for analytic functions on the polydisk.* This originates with the works of Agler [2], [1]. From the view point of system theory, one is dealing here with the conservative version of the systems known as the Roesser model or the Fornasini–Marchesini model in the multidimensional system theory literature; see [71], [46].
- *Function theory on the free semigroup and on the unit ball of* $\mathbb{C}^N$. From the view point of system theory, one considers here the realization problem for formal power series in non-commuting variables that appeared first in the theory of automata, see Schützenberger [74], [75] and Fliess [44], [45] (for a good survey see [17]), and more recently in robust control of linear systems subjected to structured possibly time-varying uncertainty (see Beck, Doyle and Glover [15] and Lu, Zhou and Doyle [59]). In operator theory, two main parallel directions may be distinguished; the first direction is along the lines of the works of Drury [43], Frazho [47], [48], Bunce [26], and especially the vast work of Popescu [65], [63], [64], [66], where various one-dimensional models are extended to the case of several non-commuting operators. Another direction is related to the representations of the Cuntz algebra and is along the line of the works of Davidson and Pitts (see [36] and [37]) and Bratelli and Jorgensen [24]. When one abelianizes the setting, one obtains results on the theory of multipliers in the so-called Arveson space of the ball (see [12]), which are closely related with the theory of complete Nevanlinna–Pick kernels; see the works of Quiggin [70], McCullough and Trent [60] and Agler and McCarthy [3]. We note also connections with the theory of wavelets and with system theory on trees; see [16], [10].
- *Hyponormal operators, subnormal operators, and related topics.* Though nominally dealing with a single operator, the theory of hyponormal operators and of certain classes of subnormal operators has many features in common with multivariable operator theory. We have in mind, in particular, the works of Putinar [68], Xia [81], and Yakubovich [82]. For an excellent general survey of the theory of hyponormal operators, see [80]. Closely related is the principal function theory of Carey and Pincus, which is a far reaching development of the theory of Kreĭn's spectral shift function; see [62], [27], [28]. Another

closely related topic is the study of multi-dimensional moment problems; of the vast literature we mention (in addition to [68]) the works of Curto and Fialkow [33], [34] and of Putinar and Vasilescu [69].

- *Hyperanalytic functions and applications.* Left (resp. right) hyperanalytic functions are quaternionic-valued functions in the kernel of the left (resp. right) Cauchy–Fueter operator (these are extensions to $\mathbb{R}^4$ of the operator $\frac{\partial}{\partial x} + i\frac{\partial}{\partial y}$). The theory is non-commutative and a supplementary difficulty is that the product of two (say, left) hyperanalytic functions need not be left hyperanalytic. Setting the real part of the quaternionic variable to be zero, one obtains a real analytic quaternionic-valued function. Conversely, the Cauchy–Kovalevskaya theorem allows to associate (at least locally) to any such function a hyperanalytic function. Identifying the quaternions with $\mathbb{C}^2$ one obtains an extension of the theory of functions of one complex variable to maps from (open subsets of) $\mathbb{C}^2$ into $\mathbb{C}^2$. Rather than two variables there are now three non-commutative non-independent hyperanalytic variables and the counterparts of the polynomials $z_1^{n_1} z_2^{n_2}$ are now non-commutative polynomials (called the Fueter polynomials) in these hyperanalytic variables. The original papers of Fueter (see, e.g., [50], [49]) are still worth a careful reading.
- *Holomorphic deformations of linear differential equations.* One approach to study of non-linear differential equations, originating in the papers of Schlesinger [73] and Garnier [51], is to represent the non-linear equation as the compatibility condition for some over-determined linear differential system and consider the corresponding families (so-called deformations) of ordinary linear equations. From the view point of this theory, the situation when the linear equations admit rational solutions is exceptional: the non-resonance conditions, the importance of which can be illustrated by Bolibruch's counterexample to Hilbert's 21st problem (see [11]), are not met. However, analysis of this situation in terms of the system realization theory may lead to explicit solutions and shed some light on various resonance phenomena.

The papers in the present volume can be divided along these categories as follows:

Polydisk function theory:

The volume contains a fourth part of the translation of the unpublished thesis [18] of Bessmertnyĭ, which foreshadowed many subsequent developments and contains a wealth of ideas still to be explored. The other parts are available in [20], [19] and [21]. The paper of Reurings and Rodman, *One-sided tangential interpolation for Hilbert–Schmidt operator functions with symmetries on the bidisk*, deals with interpolation in the bidisk in the setting of H^2 rather than of H^∞.

Non-commutative function theory and operator theory:

The first paper in this category in the volume is the paper of Ball and Vinnikov, *Functional models for representations of the Cuntz algebra*. There, the authors develop functional models and a certain theory of Fourier representation for a representation of the Cuntz algebra (i.e., a row unitary operator). Next we have the paper of Banks, Constantinescu and Johnson, *Relations on non-commutative variables and associated orthogonal polynomials*, where the authors survey various settings where analogs of classical ideas concerning orthogonal polynomials and associated positive kernels occur. The paper serves as a useful invitation and orientation for the reader to explore any particular topic more deeply. In the paper of Kalyuzhnyĭ-Verbovetzkiĭ, *On the Bessmertnyĭ class of homogeneous positive holomorphic functions on a product of matrix halfplanes*, a recent investigation of the author on the Bessmertnyĭ class of operator-valued functions on the open right poly-halfplane which admit a so-called long resolvent representation (i.e., a Schur complement formula applied to a linear homogeneous pencil of operators with positive semidefinite operator coefficients), is generalized to a more general "non-commutative" domain, a product of matrix halfplanes. The study of the Bessmertnyĭ class (as well as its generalization) is motivated by the electrical networks theory: as shown by M.F. Bessmertnyĭ [18], for the case of matrix-valued functions for which finite-dimensional long resolvent representations exist, this class is exactly the class of characteristic functions of passive electrical $2n$-poles where impedances of the elements of a circuit are considered as independent variables. Finally, in the paper *Hardy algebras associated with W^*-correspondences (point evaluation and Schur class functions)*, Muhly and Solel deal with an extension of the non-commutative theory from the point of view of non-self-adjoint operator algebras.

Hyponormal and subnormal operators and related topics:

The paper of Putinar, *Notes on generalized lemniscates*, is a survey of the theory of domains bounded by a level set of the matrix resolvent localized at a cyclic vector. The subject has its roots in the theory of hyponormal operators on the one hand and in the theory of quadrature domains on the other. While both topics are mentioned in the paper, the main goal is to present the theory of these domains (that the author calls "generalized lemniscates") as an independent subject matter, with a wealth of interesting properties and applications. The paper of Szafraniec, *Orthogonality of polynomials on algebraic sets*, surveys recent extensive work of the author and his coworkers on polynomials in several variables orthogonal on an algebraic set (or more generally with respect to a positive semidefinite functional) and three term recurrence relations. As it happens often the general approach sheds new light also on the classical one-dimensional situation.

Hyperanalytic functions:

In the paper *Operator methods for solutions of differential equations based on their symmetries*, Eidelman and Krasnov deal with construction of explicit solutions for some classes of partial differential equations of importance in physics, such as evolution equations, homogeneous linear equations with constant coefficients, and analytic systems of partial differential equations. The method used involves an explicit construction of the symmetry operators for the given partial differential operator and the study of the corresponding algebraic relations; the solutions of the partial differential equation are then obtained via the action of the symmetry operators on the "simplest" solution. This allows to obtain representations of Clifford-analytic functions in terms of power series in operator indeterminates. Luna–Elizarrarás and Shapiro in *Preservation of the norms of linear operators acting on some quaternionic function spaces* consider quaternionic analogs of some classical real spaces and in particular compare the norms of operators in the original space and in the quaternionic extension.

Holomorphic deformations of linear differential equations:

This direction is represented in the present volume by the paper of Katsnelson and Volok, *Rational solutions of the Schlesinger system and rational matrix functions II*, which presents an explicit construction of the multi-parametric holomorphic families of rational matrix functions, corresponding to rational solutions of the Schlesinger non-linear system of partial differential equations.

There are many other directions that are not represented in this volume. Without the pretense of even trying to be comprehensive we mention in particular:

- Model theory for commuting operator tuples subject to various higher-order contractivity assumptions; see [35], [67].
- A multitude of results in spectral multivariable operator theory (many of them related to the theory of analytic functions of several complex variables) stemming to a large extent from the discovery by Taylor of the notions of the joint spectrum [78] and of the analytic functional calculus [77] for commuting operators (see [32] for a survey of some of these).
- The work of Douglas and of his collaborators based on the theory of Hilbert modules; see [42], [40], [41].
- The work of Agler, Young and their collaborators on operator theory and realization theory related to function theory on the symmetrized bidisk, with applications to the two-by-two spectral Nevanlinna–Pick problem; see [5], [4], [6].
- Spectral analysis and the notion of the characteristic function for commuting operators, related to overdetermined multi-dimensional systems. The main notion is that of an operator vessel, due to Livšic; see [56], [57], [58]. This turns out to be closely related to function theory on a Riemann surface; see [79],[13].
- The work of Cotlar and Sadosky on multievolution scattering systems, with applications to interpolation problems and harmonic analysis in several variables; see [30], [31], [72].

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