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# Temporal Logic

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Arthur Prior  
In Memoriam

## General Preface to the LEP

The aim of the Library of Exact Philosophy is to keep alive the spirit, if not the letter, of the Vienna Circle. It will consequently adopt high standards of rigor: it will demand the clear statement of problems, their careful handling with the relevant logical or mathematical tools, and a critical analysis of the assumptions and results of every piece of philosophical research.

Like the Vienna Circle, the Library of Exact Philosophy sees in mathematics and science the wellsprings of contemporary intellectual culture as well as sources of inspiration for some of the problems and methods of philosophy. The Library of Exact Philosophy will also stress the desirability of regarding philosophical research as a cooperative enterprise carried out with exact tools and with the purpose of extending, deepening, and systematizing our knowledge about human knowledge.

But, unlike the Vienna Circle, the Library of Exact Philosophy will not adopt a school attitude. It will encourage constructive work done across school frontiers and it will attempt to minimize sterile quarrels. And it will not restrict the kinds of philosophical problem: the Library of Exact Philosophy will welcome not only logic, semantics and epistemology, but also metaphysics, value theory and ethics as long as they are conceived in a clear and cogent way, and are in agreement with contemporary science.

Montreal, January 1970

Mario Bunge

## Preface

This book is an introduction to temporal logic, a now flourishing branch of philosophical logic whose origin is of recent date, its main impetus having been provided by the publication in the late 1950s of A. N. PRIOR's pioneering book, *Time and Modality* (Oxford, The Clarendon Press, 1957). Virtually all work in the field to around 1966 is surveyed in PRIOR's elegant treatise *Past, Present and Future* (Oxford, The Clarendon Press, 1967). In consequence, it is no simple matter to write a comprehensive book on the subject without merely rehearsing material already dealt with in PRIOR's works. We believe, however, that the present book succeeds in this difficult endeavor because it approaches established materials from wholly novel points of departure, and is thus able to attain new perspectives and achieve new results. Its introductory character notwithstanding, the present work is consequently in substantial measure devoted to an exposition of new findings and a demonstration of new results.

Parts of the book have been published previously. Chapter II is a modified version of an article of the same title by N. RESCHER and JAMES GARSON in *The Journal of Symbolic Logic* (vol. 33 [1968], pp. 537—548). And Chapter XIII is a modified version of the article "Temporally Conditioned Descriptions" by N. RESCHER and JOHN ROBISON in *Ratio*, vol. 8 (1966), pp. 46—54. The authors are grateful to Professors GARSON and ROBISON, and to the editors of the journal involved, for their permission to use this materials here.

The authors acknowledge with thanks the conscientious assistance of JUDITH BAZY (Mrs. MARTIN STANTON) and Miss KATHY WALSH in preparing the difficult typescript.

The authors are very grateful to DOROTHY HENLE, ARNOLD VAN DER NAT, and ZANE PARKS for assistance in correcting the proofs.

Shortly after completion of the work we learned of the tragic death of ARTHUR PRIOR. It pleases us that a few days before he was able to examine the work and approve our intention to dedicate it to him.

Pittsburgh, Spring, 1971

N. Rescher and A. Urquhart



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