

**LINEAR PROGRAMMING
IN INDUSTRY
THEORY AND APPLICATIONS**

AN INTRODUCTION

BY

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WITH 6 FIGURES

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To my parents

Preface

The present volume is intended to serve a twofold purpose. First, it provides a university text of Linear Programming for students of economics or operations research interested in the theory of production and cost and its practical applications; secondly, it is the author's hope that engineers, business executives, managers, and others responsible for the organization and planning of industrial operations may find the book useful as an introduction to Linear Programming methods and techniques. Despite the different backgrounds of these categories of potential readers, their respective fields overlap to a considerable extent; both are concerned with economic optimization problems, and the use of Linear Programming to problems of production planning is simply applied theory of production. The non-economist reader may, but should not, pass over Chapter IV in which the linear production model is linked up with the economic theory of production.

Without being an advanced text, the book aims at covering enough ground to make the reader capable of detecting, formulating, and solving such linear planning problems as he may encounter within his particular field. No heavy demands are made on the reader's mathematical proficiency; except for the proofs in the Appendix—which may be skipped if desired—the mathematical exposition is purely elementary, involving only simple linear relations. In the author's experience, the pedagogical advantages of this approach, as compared with the use of matrix algebra, amply justify the sacrifice of mathematical elegance and typographical simplicity, particularly in explaining the simplex method.

The book is based on lectures given at the University of Copenhagen, the Norwegian College of Agriculture, and the University of Illinois, and on courses held at Dansk Ingeniørforening (The Institution of Danish Civil Engineers). Part of the material presented has been published previously as articles in *Erhvervsøkonomisk Tidsskrift*, *Metrika*, *Nordisk Matematisk Tidsskrift*, *Nordisk Tidsskrift for Industriel Statistik*, *Nordisk Tidsskrift for Teknisk Økonomi*, and *Operations Research*. I wish to thank the editors for permission to use this material. I am particularly grateful to Professor *David Fog* for valuable editorial criticism, to Professor *David Gale* for the proof of the Fundamental Theorem, and to Mr. *Ernst Lykke Jensen*, who outlined the inventory model of Chapter V (first published jointly in *Nordisk Tidsskrift for Industriel Statistik*, 1956).

I also wish to thank the editors and publishers of *American Economic Review*, *Econometrica*, *Harvard Business Review*, *Journal of Farm Economics*, and *Operations Research*, as well as International Business Machines Corporation, U.S.A., and the Unione Industriale di Torino, for the permission to quote from their publications, and to acknowledge my obligation to the respective authors.

The preparation of the book was made possible by a generous grant from the Graduate Research Board of the University of Illinois, to which my sincere thanks are due.

I am indebted to Miss *Mariam J. Meredith* for her careful and efficient typing of the manuscript.

Copenhagen, April, 1960

Sven Danø

Preface to Second Edition

A few misprints have been corrected in this edition and an additional reference inserted in the bibliography. Apart from this the second edition is the same as the first.

Copenhagen, May, 1963

Sven Danø

Preface to Third Edition

Apart from a few additional footnotes and references the third edition is a reprint of the second.

Copenhagen, April, 1965

Sven Danø

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