

INTERNATIONAL CENTRE FOR MECHANICAL SCIENCES

COURSES AND LECTURES - No. 17



ANGELO MARZOLLO
UNIVERSITY OF TRIESTE

CONTROLLABILITY AND OPTIMIZATION

LECTURES HELD AT THE DEPARTMENT
FOR AUTOMATION AND INFORMATION
SEPTEMBER - OCTOBER 1969

UDINE 1969



SPRINGER-VERLAG WIEN GMBH

This work is subject to copyright.

All rights are reserved,

**whether the whole or part of the material is concerned
specifically those of translation, reprinting, re-use of illustrations,
broadcasting, reproduction by photocopying machine
or similar means, and storage in data banks.**

© 1972 by Springer-Verlag Wien

Originally published by Springer-Verlag Wien New York in 1972

ISBN 978-3-211-81123-8

ISBN 978-3-7091-2959-3 (eBook)

DOI 10.1007/978-3-7091-2959-3

P r e f a c e

In Chapter I of these lecture notes functional analysis methods will be used to derive in detail the conditions of controllability of continuous linear time-varying systems, for the case in which the controls are bounded in norm by a given constant, the time interval is not only finite but also fixed, and the controllability is intended as the possibility of transferring the state vector from one given point \underline{x}_0 to an other given point \underline{x}_1 of state space. These conditions are both more general and somewhat of more practical interest than the usual controllability conditions in which initial and final points, as well as the time interval, are fixed, and the controls are not norm-limited.

From the controllability conditions, as exposed here, it is easy to derive conditions of optimality (both in time and in norm), as it will be done in Chapter II, where for the special case of controls belonging to an Hilbert space the completely explicit form of optimal controls will be given. An exercise will clarify some delicate points of the theory, particularly for the case in which the system is not controllable in the usual sense of the term, in which case time optimal controls may be not norm minimal.

The main result of Chapter III is the

formal solution of the problem of "controllability in the presence of noise", which is a problem analogous to the one of Chapter I, but with noise present (and the point \underline{x}_1 , substituted with a given region A of the state space).

Chapter I and II, as well as the first part of Chapter III are essentially an elaboration of Professor H.A. Antosiewicz's material, already appeared in [1] .

Only the formal solution of the problem of controllability in the presence of noise, which constitutes the second part of Chapter III, is original; it should be pointed out, however, that the method and the main tool used (separation theorem for convex compact sets) were inspired by Professor Antosiewicz's idea for the solution of the controllability problem in the absence of noise.

I wish to acknowledge Dr. S. De Julio for the very useful exchange of ideas on the material exposed in Chapter I and II; I wish also to thank CISM for the invitation to held this course and its students for their attention and helpful suggestions.

Angelo Marzollo

Udine, October 1969