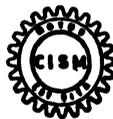


INTERNATIONAL CENTRE FOR MECHANICAL SCIENCES

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NONLINEAR ANALYSIS OF SHELLS BY  
FINITE ELEMENTS

EDITED BY

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## **PREFACE**

*This monograph is based on the lecture notes of the course "Nonlinear Analysis of Shells by Finite Elements" given at the International Centre for Mechanical Sciences (CISM) in Udine, Italy, from June 24 to 28, 1991 by Professors E.N. Dvorkin, Siderca, Buenos Aires, Argentina, O. Oñate, Universidad Politecnica de Cataluna, Barcelona, Spain, E. Ramm, Univ. Stuttgart, FRG, F.G. Rammerstorfer, Vienna Univ. of Technology, Austria, R.L. Taylor, University of California, USA, W. Wagner, Univ. Hannover, FRG, and W. Wunderlich, München Univ. of Technology, FRG.*

*Enhanced safety requirements together with the demand for reduced weight in the design of mechanical as well as civil engineering structures are leading to the development of new design concepts, to the use of advanced materials or new material combinations and to more accurate calculation methods. In many applications shell structures in combination with composite materials are replacing conventional constructions, and optimization methods become more important. Such weight saving strategies may result in rather flexible structures undergoing large deformations, and the utmost utilization of the strength of the materials requires the consideration of the materials' nonlinear behavior. In the analysis of the structural behavior those aspects can only be treated by nonlinear methods.*

*Under these aspects, it was the main objective of the course to report on recent developments in the field of stress and deformation analysis as well nonlinear stability and optimization analysis of shells by the finite element method.*

*The following topics are treated in this monograph:*

*Alternative shell element formulations in large displacement and rotation analysis comparison of shell theory based elements versus degenerated solid approach, hybrid-mixed formulations, i.e. displacement models with additional assumptions for strains or stresses, assumed strain formulations, assessment of shell elements with respect to locking phenomena, ... Furthermore, implementation and modelling aspects are discussed on the basis of the individual element formulations.*

*Finite element formulations are described for stiffened shells as well as for composite and sandwich shells under large deformations including some aspects of the material description of composites and concrete. Layered fiber-composite shells and sandwich shells as well as reinforced concrete shells are treated, and algorithms for the computation of failure models and for post-failure analysis for composite shells under mechanical and thermal loads are presented.*

*With respect to optimization of shell structures this monograph contains optimization strategies, the description of sensitivity analysis, of the design element concept and of shape finding methods of free form shells.*

*Furthermore, algorithms for the treatment of the nonlinear stability behavior of shell structures (including bifurcation and snap-through buckling) are presented in the book.*

*The theoretical considerations are accompanied by the presentation of numerical examples and practical applications.*

*It is my pleasure to thank all the colleagues who contributed to the course and this book. I also thank CISM for organizing the course and for the ospitality which the lecturers were provided with during their stay in Udine. The lecturers and authors owe special thank to Professor Sandor Kaliszky, Rector of CISM, for his efforts in supporting this course, and the Professor Carlo Tasso for encouraging the lecturers to write this monograph.*

*Franz G. Rammerstorfer*

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