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INTERNATIONAL CENTRE FOR MECHANICAL SCIENCES

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MECHANICS AND THERMOMECHANICS  
OF RUBBERLIKE SOLIDS

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## PREFACE

*Natural rubber is obtained from the milky secretion (latex) of various plants, but today the only commercial sources of natural rubber are the tree *Hevea brasiliensis* and the shrub known as guayule (*Parthenium argentatum*). *Hevea brasiliensis* is a tropical tree native to the Amazon Basin in Brazil and adjoining countries, whereas guayule is native to the arid regions of Mexico and the south west of the United States.*

*When the rubber tree is tapped we obtain latex. It has a milky colour and if it is allowed to flow onto a hand, for instance, it will quickly coagulate and form a ribbon or strand. These strands can be rubbed together to form a ball that is capable of bouncing. Unless the latex is preserved it will rapidly degrade and normally ammonia is used as a preservative. In this raw state latex is perfectly safe and can be used as an adhesive. *Hevea* latex is a polydisperse system in which rubber particles constitute 30–45% of its volume.*

*Native Americans used rubber for the production of sports equipment, as incense and lip balm, and also for the production of religious figurines. Still today Amazon Indians use it to smear over their feet to wear a kind of boot.*

*Of all the wonderful tales brought back by Christopher Columbus in 1496 after his second voyage to the New World, none was stranger than the tale of a ball used by the people of Haiti that bounced. Although they did not realize it, Columbus and his crew were the first Europeans to see rubber. The Aztecs and other Central American nations used rubber balls during a game named Chaah or Tlachtlic. The game was a mixture of soccer, volleyball and basketball in which two teams of six players attempted to hit the ball through one of the stone rings attached to a wall.*

*Rubber did not really attract the interest of Europeans until reports about it were made (1736-51) to the French Academy of Sciences by Charles de la Condamine and François Fresneau and also after this date, initially, the new material was merely a scientific curiosity. The British scientist Joseph Priestley remarked on its usefulness for rubbing pencil marks from paper in 1770. Other applications gradually developed, notably for waterproofing shoes and clothing, and the first rubber factory in the world was established near Paris in 1803, the first in England by Thomas Hancock in 1820, and in the 1830s John Haskins and Edward Chaffee established the first rubber-goods factory in the United States.*

*At the time rubber had many weaknesses. It softened with heat and hardened with cold; it was tacky, odorous, and perishable. In 1834 the German chemist Friedrich Ludersdorf and the American chemist Nathaniel Hayward discovered that the addition of sulphur to gum rubber lessened or eliminated the stickiness of finished rubber goods. In 1839 the American inventor Charles Goodyear, using the findings of the two chemists, discovered that cooking rubber with sulphur, in a process called vulcanisation, removed the gum's unfavourable properties. Vulcanized rubber has increased strength and elasticity and greater resistance to changes in temperature than*

*unvulcanized rubber; it is impermeable to gases, and resistant to abrasion, chemical action, heat, and electricity. Vulcanized rubber also exhibits high frictional resistance on dry surfaces and low frictional resistance on water-wet surfaces. The vulcanization process remains fundamentally the same as it was in 1839. With the invention of the bicycle, the motorcar and the pneumatic rubber tire, demand for rubber grew rapidly. By 1900 more than 40,000 tons were used each year. Today the world's natural rubber use is over 4,500,000 tons per annum.*

*There are also artificially produced substances that resemble natural rubber in essential chemical and physical properties. Such substances, usually called elastomers, are produced by chemical reactions, known as condensation or polymerization, of certain unsaturated hydrocarbons. The basic materials of synthetic rubber are monomers: compounds of relatively low molecular weight that form the building units of huge molecules called polymers. After fabrication elastomers are also cured by vulcanization. The origin of synthetic-rubber technology can be traced back to 1860, when the British chemist Charles Hanson Greville Williams (1829–1910) determined that natural rubber was a polymer of the monomer isoprene. Much effort was directed during the next 70 years to synthesizing rubber in the laboratory by using isoprene as the monomer. Other monomers were also investigated, and during World War I German chemists polymerized dimethylbutadiene to produce an elastomer called methyl rubber, which was of limited use. A breakthrough in synthetic-rubber research did not occur, however, until about 1930, when the American chemist Wallace Hume Carothers (1896–1937) and the German scientist Hermann Staudinger (1881–1965) placed polymer chemistry on a theoretically sound scientific basis. The work of these scientists contributed greatly to the present-day knowledge that polymers are huge molecules with chain-like structures, arising from chemical reactions of large numbers of monomers, and that elastomers can be prepared from monomers other than isoprene. More than a dozen major classes of synthetic rubber are made from raw material derived from petroleum, coal, oil, natural gas, and acetylene. Many of them are copolymers, i.e. polymers consisting of more than one monomer. By changing the composition it is possible to achieve particular properties desired for specific applications. Today 70% of all rubber used is synthetic, but despite this fact natural rubber continues to be in high demand due to its better elasticity, resilience and heat transfer properties.*

*The first experiments on the elasticity of natural rubber were carried out by John Gough in 1802 and the results were published in the Memoirs of the Literary and Philosophical Society of Manchester, Second Series (Volume I, 1805, 288–295). The title of this paper was: 'A description of a property of caoutchouc or Indian rubber with some reflections on the cause of elasticity of this substance'. Two experiments were conducted: the first concerned the evolution of heat on stretching rubber; the second the counterintuitive effect of contraction of stretched rubber upon heating. Other experiments had been done previously to determine the chemical composition of rubber. In 1930 the polymer physicists W. Kuhn, E. Guth and H. Mark developed the first mathematical model to explain the elasticity of rubber. Progress in the quantitative modelling of polymeric materials was made mainly in the period from the 1930s to the 1960s by polymer chemists such as P.J. Flory and L.R.G. Treloar, but also by applied mathematicians, notably R.S. Rivlin. The approach used by polymer chemists and physicists is usually based at the molecular*

*level, whereas applied mathematicians adopt a phenomenological approach based on the theories of continuum mechanics and nonlinear elasticity.*

*Today interest in rubber-like materials it is still very much alive. Rubber is used mainly to produce tires (68%), but also to manufacture other goods and products; for example, footwear (5%), engineering devices (7.8%), latex products (8%), adhesives (3.2%) and pharmaceutical articles, floor covering, sports goods and toys. The development of special 'smart' rubber-like materials such as electro-active or magneto-sensitive rubbers has introduced the possibility of new and exciting applications; for example, active suspensions for vehicles and artificial muscles. Moreover, the mechanics of rubber-like materials have some similarities to those of soft tissue (the materials of arteries, ligaments, tendons, etc.). The biomechanics of soft tissues is a multidisciplinary field of research, but it requires a good background of knowledge from our experience of analysing the mechanics of rubber-like materials.*

*Within this framework it is very important to have a complete and rational approach to the mathematical modelling of the mechanics of rubber-like materials. Models for rubber-like materials may be based on the molecular or the phenomenological approach. The aim of CISM advance course on the Mechanics and Thermomechanics of Rubberlike Solids, held in Udine during the week September 30 to October 4, 2002, was to provide a unique state-of-the art multidisciplinary overview of rubber mechanics through the interactions of topics ranging from the mathematical modelling of material response (elastic and inelastic) at both the molecular and phenomenological levels, analysis of the governing differential equations and boundary-value problems, computational methods for the solution of these problems and experimental techniques for the determination of the mechanical properties of rubber, with particular reference to industrial applications and the needs of industry for new developments in material modelling and analysis.*

*The course included seven lectures by Burak Erman (Sabanci University, Turkey) on the classical molecular theories of rubber elasticity and bioelastomers, and six lectures by Paul Buckley (University of Oxford, UK) on experimental techniques. Giuseppe Saccomandi (University of Lecce, Italy) devoted five lectures to the phenomenological theory of nonlinear elasticity, and Ray Ogden (University of Glasgow, UK) delivered five lectures on inelastic theories, including pseudoelasticity and viscoelasticity. Numerical methods were discussed in seven lectures by Sanjay Govindjee (University of California at Berkeley, USA), while some industrial applications were highlighted in three lectures by Franco Bettinali (ENEL s.p.a., Italy). The course was attended by 35 students with different background (engineering science, material science, mathematics and physics), from all over the world, both from universities and from companies in the rubber industry. The students contributed to a very interesting discussion afternoon with several short talks on topics ranging from fatigue of rubber to the mechanics of elastic structures.*

*As always the CISM staff and the Rector M.G. Velarde have been very helpful and professional in coping with all the technical needs of the course. We think that the lecture notes provide, not only a*

*bird's eye overview of this interdisciplinary subject but also show that the subject remains very active and offers many interesting challenges from the perspectives of both pure and applied research.*

*Giuseppe Saccomandi  
Ray Ogden*

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