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INTERNATIONAL CENTRE FOR MECHANICAL SCIENCES

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NEW DEVELOPMENTS
IN CONTACT PROBLEMS

EDITED BY

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UNIVERSITY OF HANNOVER

PANAGIOTIS PANAGIOTOPOULOS[†]
ARISTOTLE UNIVERSITY OF THESSALONIKI



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**Panagiotis D. Panagiotopoulos
1950-1998**

With the unexpected death of Panagiotis (Panos) Panagiotopoulos, the Mathematics and Mechanics community has lost one of its most prominent researchers. During the editorial work related to this book, unfortunate news came that my co-editor, Professor Panos Pangiotopoulos, died unexpectedly on August 12, 1998. He is survived by his wife, son and daughter, as well as his parents.

Professor Panagiotopoulos held the chair of steel structures at Aristotle University, Thessaloniki, Greece. Panos Panagiotopoulos was born in Thessaloniki in 1950. He graduated in 1972 from the Department of Civil Engineering of the Aristotle University in Thessaloniki. He received his doctoral degree in Civil Engineering, with the academic title Dr. Ing., only two years later at the same university. Afterwards, he won an Alexander von Humboldt Research Fellowship by which he stayed in Germany, at the RWTH Aachen, until 1977. In addition, in 1977 he completed a "Habilitation" thesis at the

Faculty of Mathematics and Physics of RWTH and became "Dozent" at this faculty. After returning to Greece, in 1978, due to his outstanding scientific achievements, Panos Panagiotopoulos was appointed Full Professor and Director of the Institute of Steel Structures at the Department of Civil Engineering at Aristotle University in Thessaloniki, *at the age of only 28*.

Starting in the mid seventies, Panos pioneered a number of techniques in mechanics and mathematical analysis. His research was especially devoted to the mechanics of unilateral problems, an area of Nonsmooth Mechanics involving nondifferentiable or variational inequalities. In 1982, he coined the term "hemivariational inequalities", which extended the realm of unilateral problems from convex to non-convex cases. Within this field, he had created his own unique style. Panos' research interests were extraordinarily broad. He strove to combine mathematics and mechanics in order to develop new tools for the mathematical analysis of unilateral problems, and also numerical methods for computer simulations of a large range of engineering applications. Towards the end of his career, a natural extension of his research interests was from unilateral problems in statics to dynamic cases, including impact.

During his scientific career Panos guided over 25 Ph. D. students. He inspired many researchers and initiated new developments in the mechanics of unilateral problems. Due to his prolific scientific achievements - he published over 250 papers and 9 books - he attracted scholars from all over the world to his institute in Greece.

Panos had a worldwide reputation, and was repeatedly invited to give keynote lectures at virtually every important conference in his field. He has been elected in 1981 as Honorary Professor of Mechanics, at the Faculty of Mathematics and Physics, in RWTH Aachen and offered a position as Full Professor at the same university in 1985, but he chose to stay in Thessaloniki. Professor Panagiotopoulos was visiting Professor at the University of Hamburg, Germany, at the Massachusetts Institute of Technology, United States, at Pontificia Universidade Catolica, Rio de Janeiro, Brasil and at the University of Namur, Belgium. He was member of the editorial boards of ten of the leading journals in his field of research. He became Corresponding Member of the Academy of Sciences, in Athens, Greece and in 1989 Full Member of the Academia Europaea, London, England. In 1995 he was awarded one of the highest scientific honors in Italy, the International *Agostinelli* prize of the Accademia Nazionale dei Lincei, Rome.

In the international scientific community, all who met Panos were fascinated by his charming personality, his great sense of humour, and, not in the least, his outstanding scientific work. His research was profound and will have lasting impact. The research community will certainly remember him as one of the most outstanding researchers in the field of unilateral problems. Colleagues and friends from all over the world will never forget Panos Panagiotopoulos and will deeply regret losing him.

Peter Wriggers

PREFACE

The study of contact problems in Mechanics is associated with the formulation and treatment of variational inequalities. This topic has led to developments of new and interesting areas in Mechanics, Applied Mathematics and the Engineering Sciences during the last three decades. New insight into contact problems has been gained, on the theoretical side, by the application of variational inequalities and linear complementarity problems in Mechanics. Also, this has led to the development of the theory of hemivariational inequalities for nonconvex energy functions. Furthermore, numerical methods to solve large two and three dimensional contact problems, have become increasingly important in the engineering practice.

Many researchers have developed innovative methodologies in the recent years. This made the subject of Contact Mechanics a mature area. With the goal to bring some of the new developments to a wider audience, the International Center for Mechanical Sciences in Udine had organized a course on “Contact Problems: Theory, Methods and Application” in 1997. Lectures have been presented by scientists from France, Greece, Germany, Sweden and Switzerland. The contributions collected in this volume summarize the lectures presented during the course.

The book consists of four chapters. The first chapter is devoted to the basic formulation of unilateral contact problems in Mechanics using the modern concept of nonsmooth analysis. This chapter covers general principles and tribological laws. The second chapter is concerned with the treatment of contact and friction of discrete mechanical structures through mathematical programming techniques. It contains mainly three parts which deal with the mathematical modelling of discrete finite dimensional frictional problems, the formulation of contact as a linear complementary problem, and structural optimization in the presence of contact constraints. The quasistatic Signorini problem is investigated

in the third chapter in the presence of Coulomb friction and is coupled to adhesion. The last chapter is subdivided into two sections. The first lays the foundation for thermomechanical contact problems in the presence of finite deformations. In this vein the finite element method is formulated. The second section is devoted to adaptive finite element techniques for contact problems with and without friction.

During the course of the editorial work related to this book, my co-editor Professor P. D. Panagiotopoulos died unexpectedly. Sadly, due to this tragic loss, it was not possible to include his valuable contribution on hemivariational inequalities, with application to problems of Contact Mechanics, in this volume.

It is hoped that the reader will obtain an overview on formulation, mathematical analysis and numerical solution procedures of contact problems. In this respect the book should be of value to applied mathematicians and engineers who are concerned with Contact Mechanics.

The support of the CISM staff and efficient and smooth organization during the course in Udine is greatly appreciated. Furthermore, I thank Prof. C. Tasso in his capacity as editor of this series for his patience.

*Peter Wriggers
Panagiotis Panagiotopoulos*

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