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MULTICRITERIA DECISION
MAKING

EDITED BY
G. LEITMANN
UNIVERSITY OF CALIFORNIA, BERKELEY
A. MARZOLLO
UNIVERSITY OF TRIESTE



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LIST OF CONTRIBUTORS

- A. Blaquière** Laboratoire d'Automatique Théorique,
Université de Paris VI, Paris, France
- G. Castellani** Department of Mathematics,
Ca' Foscari University, Venice, Italy
- G. Leitmann** Mechanical Engineering Department,
University of California at Berkeley,
California, USA
- Y. Medanic** Mihailo Pupin Institute, Belgrade,
Yugoslavia
- A. Marzollo** Electrical Engineering Department,
University of Trieste and International Centre for Mechanical
Sciences, Udine, Italy
- W. Stadler** Mechanical Engineering Department,
University of California at Berkeley,
California, USA
- W. Ukovich** Electrical Engineering Department,
University of Trieste, Italy
- M. Volpato** Department of Mathematics,
Ca' Foscari University, Venice, Italy
- P.L. Yu** Graduate School of Business,
University of Texas, Austin, Texas, USA.

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PREFACE

A considerable amount of research has been devoted recently to Multicriteria Decision Making, stimulated by the vast number of real problems, for example in industrial, urban and agricultural economics, in the social sciences, and in the design of complex engineering systems, where many decision makers are present or many, possibly conflicting objectives should be taken into account in order to reach some form of optimality.

A rough division into two classes may be made among the approaches to Multicriteria Decision Making problems. The first one deals mainly with the empirical determination of preference structures in some specific problems, and seeks methods for their meaningful aggregation in order to arrive, often by ad hoc and iterative procedures, at practically reasonable solutions. The lines of thought which are followed and the used methods may be looked upon as modern developments of operations research.

The second one, predominantly treated by researches whose background is often rooted in systems and control theory, or in mathematical programming and in its applications, seems more directed toward general and rigorous formulations in order to reduce Multicriteria Decision Problems conceptually to clearly defined classes of optimization problems for which definite solutions algorithms are sought.

The contributions to the present volume follow mainly the latter line of thought, although references and comparisons are made to other, sometimes non-empirical methods, for example by P.L. Yu and, in general, algorithmic solutions are proposed to specific problems as a result of the conceptual methods used. The reader is introduced to the large class of multicriteria, multiagent problems which may be treated in the framework of game theory, both for static and dynamical systems, in the first two contributions by G. Leitman and A. Blaquièrè. The formulation of the latter is so general as to encompass as specific cases the great majority of multi-objective, multiplayer problems that one may think of in cooperative, non-cooperative or mixed situations.

J. Medanic gives an exhaustive solution, both in deterministic and stochastic cases, to the optimal regulator problem with vector valued quadratic performance, and applies the developed concepts to a multiplant cooperative control problem.

The paper by W. Stadler imbeds both the static and dynamical vector optimization problem in the framework of preference optimality by borrowing techniques which have been developed mainly by mathematical economists, and so is able to give interesting sufficient and necessary conditions for optimality; vector optimization concepts are then applied to the design of minimally disturbing measuring devices for optimally controlled systems, and for optimal structural design in mechanics. A complete treatment of Domination Structures and Non-Dominated Solutions with an example of application to stock market behaviour is given by P.L. Yu.

A Marzollo and W. Ukovich discuss some basic principles underlying the concepts of Vector Optimality and then give precise conditions, using the weakest hypotheses on the involved functions, for the characterization of "weakly" "ordinarily" and "strictly" Non Dominated Decisions, in the global, local and "differential" version.

Specific economic relevance is stressed in the contribution of M. Volpato, who deals with the

optimal choice for the relative amounts of a specific good to be produced in various countries of a community in order to give the maximal community yield from the residual resources. Unlike the classical theory on the subject, non-linear conversion prices from one product to another are also considered. The problem may be solved explicitly for a rather general class of functions by using non-linear and dynamic programming techniques which are developed in the following contribution by G. Castellani. It is shown that in this framework the optimal production policy for the community is also economically optimal for each individual country.

This sketch of the contents of this volume is by no means exhaustive of the various philosophical approaches to Multicriteria Optimization contained therein, nor of the techniques which, as a consequence, are suggested for the solution of many varied problems. We express our hope that the volume as a whole will stimulate the reader to giving further thought to the conceptual and mathematical challenges offered by the present extension of optimality theory and provide him with some useful methods for solving problems in which different points of view are to be considered, or, to recall the title of this volume, to solve "Multicriteria Decision" problems.

George Leitmann

Angelo Marzollo