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# International Centre for Mechanical Sciences

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Francisco Chinesta · Pierre Ladevèze  
*Editors*

Separated Representations  
and PGD-Based  
Model Reduction  
Fundamentals and Applications



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*Editors*

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## PREFACE

*Today, in spite of the impressive progress which has been achieved in the past decade in modeling, numerical analysis, discretization techniques and computer science, many science and engineering problems remain inextricable because their numerical complexity or the restrictions imposed by the operational requirements (for example, real-time computing on deployed platforms) make it too expensive to solve them using today's technologies. These include a group of problems with no apparent relation to one another, but which can be treated in a unified manner, as this course will show. What these problems have in common is our inability or our insufficient knowledge to solve them numerically in a direct, traditional way. Thus, in order to get a solution, it is necessary to resort to some sort of model order reduction.*

*We begin this course popular by describing the construction of reduced models through a review of Proper Orthogonal Decomposition (POD) and reduced-basis models, including their mathematical foundations and some challenging applications. Then, we move to a new generation of simulation strategies based on the use of separated representations (space-parameters, space-time, space-time-parameters, space-space, ...), which have led to what is known as Proper Generalized Decomposition (PGD) techniques. Because such representations enable one to do away with dimensionality restrictions, the models can be enriched by treating parameters as additional coordinates, leading to fast and inexpensive online calculations based on richer offline parametric solutions. Separated representations are analyzed in detail in the course, from their mathematical foundations to their most spectacular applications. We illustrate how such an approximation could evolve into a new paradigm in computational science, enabling one to circumvent various computational issues in a vast array of applications in engineering science.*

*The course is intended for doctoral students, junior or senior researchers and practicing engineers working in the area of simulation software, who are confronted with the severe limitations of standard simulation techniques in the resolution of complex models.*

*We would like to thank our esteemed colleagues and friends Elias Cueto, Antonio Huerta, Boris Khoromskij and Gianluigi Rozza for*

*accepting our invitation to lecture in our course and contributing very significantly to the lecture notes (except for B. Khoromskij due to health problems).*

*We would also like to express our thanks to CISM, especially to the Rectors and staff, for their help and cooperation in organizing the course and printing the lecture notes.*

*Francisco Chinesta and Pierre Ladeveze*

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