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Maurizio Angelillo
Editor

Mechanics of Masonry Structures

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Editor

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PREFACE

It is maybe a trivial remark saying that the vast majority of masonry structures (excluding tall towers) exhibit an extraordinary stability under the effect of age and settlements and even under the repeated action of strong winds and heavy earthquakes. Someone may be skeptical about this statement since, lately, collapses of masonry structures are not so infrequent. The point is that, almost invariably, failure is caused by some unaware but diabolic alteration of the natural and pacific equilibrium of masonry. The reason of this pacific stability stems from the so called strength by shape that is typical also of other structures carrying axial forces, such as a cord or a membrane, that is unilateral structures. If pacific-unilateral stability for masonry, does not mean plainly dumb or boring stability, this is actually due to the curved structural elements (arches, vaults and domes) that started to appear systematically in masonry Architecture since the ancient Rome. The unilateral model, which appears as the clue of structural interpretation behind the design of the great Architecture masterpieces of the past, was first rationally introduced in the scientific community by Heyman in 1966, with his mile-stone paper the stone skeleton*. Since then it has been the Italian school of Structural Mechanics to carry the torch of the old masonry tradition, with the contribution of a number of individuals dragged by the charismatic leaderships of Salvatore Di Pasquale, and of whom Lucchesi, Šilhavý and myself are, in some sense, modern followers. The fire of the unilateral model, still burning in Naples in the late seventies, was poked by the unlucky event of the Irpinia earthquake of 1980. At that time I was a young Architect working under the guidance of Giovanni Castellano (a friend and former co-worker of Di Pasquale), and I had the occasion not only to eyewitness the, sometimes turbulent, discussions on the No Tension model for masonry, but also to see the model at work in the wounded body of many masonry buildings and monuments of Naples and of its battered neighbourhoods. But how comes that the unilateral model for masonry, that has been part of the traditional scientific heritage since Mery divulgated the thrust line approach of Moseley in 1840, had to be rediscovered again (and with scant success) in the second half of the twentieth century? Indeed, though the traditional unilateral approach to masonry equilibrium has had an outstanding mentor and*

divulgator in the person of Jaques Heyman, who, after writing the mentioned paper, in 1995 published a crystalline book with the same inspiring title (a book in which the author succeeds in explaining the stone behaviour to the stones themselves by using barely a few equations), it seems that the message of the traditional masonry design has not been welcomed by the modern structural engineers. A reason for this state of affairs is given by Santiago Huerta, in his paper by the provoking title Galileo was wrong *"... any engineer or architect with some formation in structural theory feels more comfortable within the frame of the strength approach of Galileo and the classical theory of structures. It requires an effort, and some study, to overcome our own prejudices and to accept that, for example, the medieval master masons, knowing nothing of mathematics, elastic theory and strength of materials, had a deeper understanding of masonry architecture than we engineers and architects of the twenty-first century do."*

The presentation given by Lucchesi, Šilhavý and myself in the first part of this book represents a modern update of the unilateral model for masonry and a step forward toward the goal of obtaining a useful practical tool for the analysis of masonry structures.

Though we believe that the unilateral model can be useful to practitioners and applied engineers, since it captures the essence of masonry mechanics, still the limits of such a crude model are apparent and there are aspects of masonry behaviour that need to be understood such as damage, degradation, friction, heterogeneity and particularly the role of the interface behaviour in the overall response of masonry. In order to appreciate the limits of validity of the simplified unilateral approach, it is important to study and interpret the experimental results with the "eyes" of more sophisticated models. Actually, all I have said until now refers to the phenomenological modelling of old masonry for which the assessment of the material properties in the detail required by fancy models is virtually impossible. The case of new masonries for which the nature of the blocks and of the mortar and of their arrangement is known and reliable, is a complete different story. A typical case is that of brick-works studied in the present book by Lebon, Sacco and Lourenco & Milani. In the end, these new masonry structures are nothing else than composite structures to which sophisticated techniques of homogenization can be applied. The theoretical and experimental study of these peculiar structures with this

more in depth focus, is not only useful for the closer simulation of their mechanical behaviour, but can put light on the mechanical phenomena that are behind the crude approximations of the Heyman's model, namely the unilateral and the no-sliding assumptions. Unilaterality is an extreme approximation for the brittleness of the material under tensile loads, brittleness being responsible for the softening behaviour of masonry at the macroscopic level. No-sliding is equivalent to assume infinite friction, and friction and sliding are the basic mechanisms in brick-brick, and brick-mortar-brick interactions. Understanding toughness and friction is then obviously a necessary step toward the goal of obtaining a detailed masonry description. Anyone working at some depth in material engineering knows that fracture and friction are still the most difficult challenges of modern Mechanics; the main strength of the simplified unilateral model of Heyman which assumes zero toughness and infinite friction is indeed its ability, while excluding these two tough guys, of being still able to make sound predictions on masonry behaviour.

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