

Dietrich Braess
Wolfgang Hackbusch
Ulrich Trottenberg (Eds.)

**Advances in
Multi-Grid Methods**

Notes on Numerical Fluid Mechanics

Volume 11

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Dietrich Braess
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Advances in Multi-Grid Methods

Proceedings of the conference held in
Oberwolfach, December 8 to 13, 1984



Springer Fachmedien Wiesbaden GmbH

CIP-Kurztitelaufnahme der Deutschen Bibliothek

Advances in multi-grid methods: proceedings of the conference held in Oberwolfach, December 8–13, 1984 / Dietrich Braess ... (ed.). — Braunschweig; Wiesbaden: Vieweg, 1985.
(Notes on numerical fluid mechanics; Vol. 11)
ISBN 978-3-528-08085-3

NE: Braess, Dietrich [Hrsg.]; GT

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Ursprünglich erschienen bei Friedr. Vieweg & Sohn Verlagsgesellschaft mbH, Braunschweig 1985

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ISBN 978-3-528-08085-3

ISBN 978-3-663-14245-4 (eBook)

DOI 10.1007/978-3-663-14245-4

PREFACE

During the week of December 8-13, 1984, a conference on *Multi-Grid Methods* was held at the Mathematisches Forschungsinstitut (Mathematical Research Institute) in Oberwolfach. The conference was suggested by the GAMM-Committee "Effiziente numerische Verfahren für partielle Differentialgleichungen". We were pleased to have 42 participants from 12 countries.

These proceedings contain some contributions to the conference. The centre of interest in the more theoretical contributions were exact convergence proofs for multi-grid method. Here, the theoretical foundation for the application of the method to the Stokes equations, the biharmonic equation in its formulation as a mixed finite element problem and other more involved problems were investigated. Moreover, improvements and new attacks for getting quantitative results on convergence rates were reported.

Another series of contributions was concerned with the development of highly efficient and fast algorithms for various partial differential equations. Also in this framework, the Stokes and the biharmonic equations were investigated. Other lectures treated problems from fluid mechanics (as Navier-Stokes and Euler equations), the dam-problem and eigenvalue problems.

The editors would like to thank Professor M. Barner, the director of Mathematisches Forschungsinstitut Oberwolfach for making this conference possible.

D. Braess, Bochum W. Hackbusch, Kiel U. Trottenberg, St. Augustin

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