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**Robust and Intensive
Design of Multivariable
Feedback Systems**

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– Multimodel Design –**

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Preface

The object of this monograph is to present some optimal design methods in a certain sense for obtaining linear multivariable feedback systems such that they become insensitive or robust properties. The treatment is mainly confined to linear time-invariant discrete systems with the exception of the sections 3.1 and 3.3 in which it was used linear time-invariant continuous systems.

Sensitivity and robust methods for multivariable feedback systems have been extensively studied about the last decade and the successful development has led to a much better understanding of the performance of insensitive and robust procedures. The contribution of this monograph is an improved behaviour of the feedback systems under the influence of disturbance, parameter variations and/or nonlinear effects with new design methods. Furthermore, in chapter 3 the reader finds improved necessary and sufficient conditions for robust stability of the closed-loop system.

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Berlin, February 1986

Irmfried Hartmann
Werner Lange
Rainer Poltmann

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Notations

\mathbb{R}^n	n-dimensional vector space
$\underline{0}$	zero vector, zero matrix
\underline{i}_v	v-th unit vector (chapter 2)
\underline{I}_n	n-series unit matrix (chapter 1 and 2)
\underline{E}_n	n-series unit matrix (chapter 3)
$\text{adj}(\underline{A})$	adjunct matrix of \underline{A}
$\det(\underline{A})$	determinant of \underline{A}
System denotation:	
n	system order
r	number of inputs
p	number of outputs
$\underline{G}(s)$	transfer function in the s-domain
$\underline{G}(z)$	discrete transfer function
$\Delta \underline{G}$	difference between two transfer functions
$\Delta(z), N(z)$	characteristic polynomial; denominator polynomial
$Z(z)$	numerator polynomial (in general)
$\underline{K}(s), \underline{F}(s)$	controller transfer function
$\underline{A}, \underline{\Phi}$	systemmatrix-continuous, discrete-
$\underline{B}, \underline{H}$	input matrix-continuous, discrete-
\underline{C}	output matrix
\underline{D}	feed through matrix
$\Delta \underline{A}$	difference between two dynamic matrices
\underline{k}_i	controller parameter vector
u, \tilde{u}	input variable
\underline{u}	vector of input-data
\underline{x}	state vector
\underline{y}	output vector

Nominal command behavior (chapter 2):

$k_i^{(j)}$	elements of a row of the transformed state gain matrix
$\Delta_R(z)$	characteristic polynomial of the nominal command behavior
$\Gamma_R(z)$	polynomial matrix designating the nominal command behavior

Controller (chapter 2):

m	order of a partial controller
$\Delta_B(z) = z^m + \beta_{m-1}z^{m-1} + \dots + \beta_1z + \beta_0$	characteristic polynomial of a partial controller
$k_i^{*(j)}$	free parameters of a partial controller which do not influence the nominal command behavior

Feedback system:

$\underline{S}(z)$	sensitivity function
$\underline{\Gamma}_C(z), \underline{Z}_C(z)$	transfer functions designating the feedback system behavior (chapter 2)

Special notation in chapter 3:

a_i	coefficient of denominator-polynomial
\underline{a}	vector of denominator polynomial coefficients
$\underline{\tilde{A}}$	sect.3.1.6: dynamic-matrix after an iteration step
b_i	coefficient of numerator-polynomial
\underline{b}	vector of numerator polynomial coefficients
d	sect.3.1: pole-distance sect.3.3.6: constant in bilinear transformation
dp	sect.3.1: distance between poles for permutation
$e(v)$	controller input-data
\underline{H}	sect.3.1.3: parameter-matrix sect.3.2.5: input-matrix

m	number of variation-cases
p	complex variable in the p-domain
$s_{i,j}$	controllability-coefficient of pole i and input j
T	sect.3.2: sampling period
\underline{T}	sect.3.2: eigenvector-matrix sect.3.3: closed-loop transfer-function
\underline{U}	matrix of input-data
\underline{v}	vector with poles
ΔV	difference between two dynamic-systems
\underline{W}	closed-loop perturbation transfer-function
\underline{w}_p	vector with permutated poles
\underline{x}_i	eigenvector; in sect. 3.2.5 eigenvalues
z	complex variable in the z-domain
\underline{z}	vector with powers of z
γ, γ_i	weighting-factor; in sect.3.2.5 eigenvalues
$\underline{\Gamma}, \hat{\underline{\Gamma}}$	weighting-matrices
$\epsilon(v)$	difference-data
ϵ	sect. 3.3: variation $0 \leq \epsilon \leq 1$
\mathcal{S}	sampling-time
$\underline{\theta}_i, \underline{\theta}$	parameter-vector, parameter-matrix
$\underline{\Phi}_c$	dynamic-matrix of controller
$\underline{\Phi}_s$	dynamic-matrix of plant
$\underline{\Phi}_T$	dynamic-matrix, low sampling period
$\underline{\psi}$	data-vector