The Universe of Quadrics

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Preface

In 2016, the authors published the first of the two intended books about curves and surfaces of degree two, "The Universe of Conics". The plan was to come up with the second volume about *surfaces* of degree two "as soon as possible".

The ulterior motive was to write two compendia containing important geometric knowledge that seems in danger of getting lost. The curves of degree two have been of interest to mathematicians ever since the advent of geometry, and the ancient Greeks already had a better understanding of them than many aspiring mathematics students nowadays. In the course of centuries, many famous mathematicians contributed to this body of knowledge. Just to name one of them, the ingenious Blaise Pascal published his "Essay pour les Coniques" at the age of sixteen. The heydays of these simplest algebraic curves might have been in the 17^{th} century, probably inspired by the fact that conics appear regularly in the universe as orbits of planets and other celestial bodies. Isaac Newton used his profound understanding of those curves when he proved Kepler's laws, which had only been conjectures until that point.

In the following centuries, mathematics became more advanced – mainly through the introduction of infinitesimal calculus. Famous mathematicians shifted their focus onto logical generalization. Carl Friedrich Gauss, who had previously achieved fame by predicting the re-appearance of the dwarf planet Ceres, transferred the remarkable properties of conics into space and found wonderful examples for his famous theorems about Differential Geometry. The 19^{th} century (Chasles, Dupin, Monge, Cayley, Klein, and many others) was probably the time that saw the most advances in knowledge about the simplest algebraic surfaces – the quadrics.

To come back to our attempt to write compendia about conics and quadrics: During the work on the first book, we soon figured out that there is so much knowledge about conics that even a thousand pages would not have been enough to cover the entire topic (the book has "only" some 500 pages). And – alas – an additional 500 pages cannot cover the entire body of knowledge about quadrics. Still, three years after the release of the first book, we think that we have collected a lot of theorems that might "stay alive" by means of this compendium. Some of the theorems may even be new, although they are just the consequence of what has been accumulated by others in the course of the time. Anyway, the saying “Good things come to those who wait” can be applied to the result. As probably most book writers can confirm, “it was much more work than expected”. Eventually, however, it is very satisfying to see how essential the surfaces of degree two are for the entire framework of mathematics (see Figure 0.1), which is based on the profound knowledge about the most elementary surfaces in space.

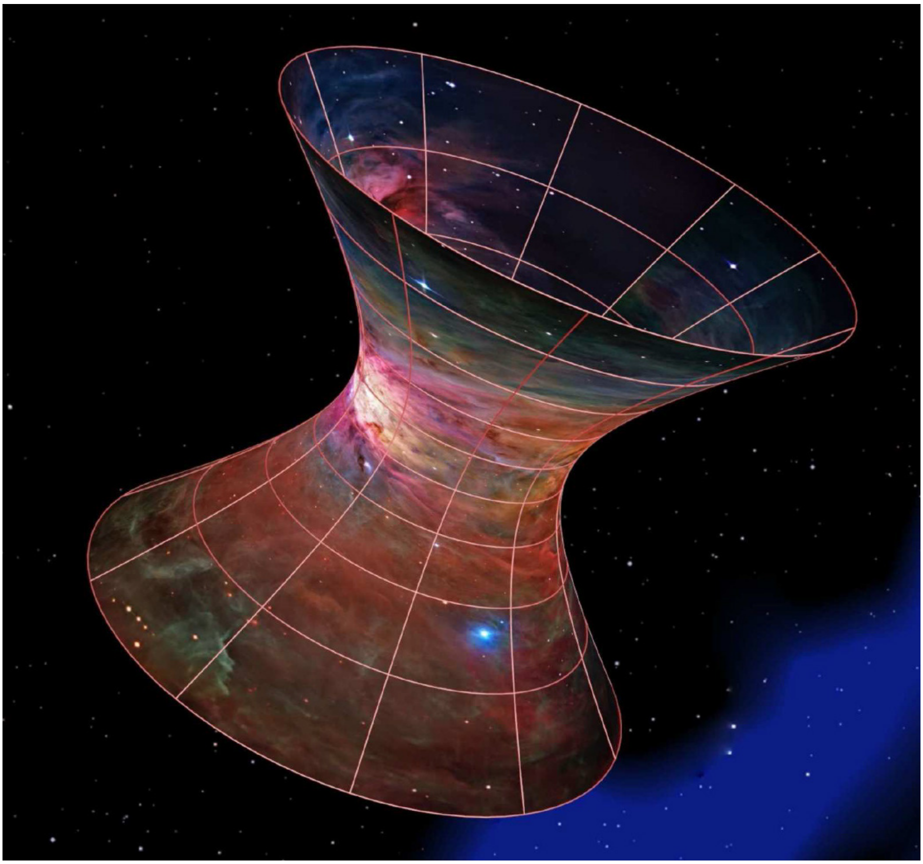

FIGURE 0.1. Hyperboloid in space – textured with space, symbolizing that the quadrics are essential when one deals with non-Euclidean geometry, Projective Geometry, Algebraic Geometry, Differential Geometry, higher dimensional geometry, and many other part disciplines of mathematics.

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