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Mathematical Analysis II

Second Edition

Springer

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Prefaces

Preface to the Second English Edition

Science has not stood still in the years since the first English edition of this book was published. For example, Fermat's last theorem has been proved, the Poincaré conjecture is now a theorem, and the Higgs boson has been discovered. Other events in science, while not directly related to the contents of a textbook in classical mathematical analysis, have indirectly led the author to learn something new, to think over something familiar, or to extend his knowledge and understanding. All of this additional knowledge and understanding end up being useful even when one speaks about something apparently completely unrelated.[1]

In addition to the original Russian edition, the book has been published in English, German, and Chinese. Various attentive multilingual readers have detected many errors in the text. Luckily, these are local errors, mostly misprints. They have assuredly all been corrected in this new edition.

But the main difference between the second and first English editions is the addition of a series of appendices to each volume. There are six of them in the first and five of them in the second. So as not to disturb the original text, they are placed at the end of each volume. The subjects of the appendices are diverse. They are meant to be useful to students (in mathematics and physics) as well as to teachers, who may be motivated by different goals. Some of the appendices are surveys, both prospective and retrospective. The final survey contains the most important conceptual achievements of the whole course, which establish connections between analysis and other parts of mathematics as a whole.

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[1]There is a story about Erdős, who, like Hadamard, lived a very long mathematical and human life. When he was quite old, a journalist who was interviewing him asked him about his age. Erdős replied, after deliberating a bit, "I remember that when I was very young, scientists established that the Earth was two billion years old. Now scientists assert that the Earth is four and a half billion years old. So, I am approximately two and a half billion years old."

I was happy to learn that this book has proven to be useful, to some extent, not only to mathematicians, but also to physicists, and even to engineers from technical schools that promote a deeper study of mathematics.

It is a real pleasure to see a new generation that thinks bigger, understands more deeply, and is able to do more than the generation on whose shoulders it grew.

Moscow, Russia V. Zorich
2015

Preface to the First English Edition

An entire generation of mathematicians has grown up during the time between the appearance of the first edition of this textbook and the publication of the fourth edition, a translation of which is before you. The book is familiar to many people, who either attended the lectures on which it is based or studied out of it, and who now teach others in universities all over the world. I am glad that it has become accessible to English-speaking readers.

This textbook consists of two parts. It is aimed primarily at university students and teachers specializing in mathematics and natural sciences, and at all those who wish to see both the rigorous mathematical theory and examples of its effective use in the solution of real problems of natural science.

The textbook exposes classical analysis as it is today, as an integral part of Mathematics in its interrelations with other modern mathematical courses such as algebra, differential geometry, differential equations, complex and functional analysis.

The two chapters with which this second book begins, summarize and explain in a general form essentially all most important results of the first volume concerning continuous and differentiable functions, as well as differential calculus. The presence of these two chapters makes the second book formally independent of the first one. This assumes, however, that the reader is sufficiently well prepared to get by without introductory considerations of the first part, which preceded the resulting formalism discussed here. This second book, containing both the differential calculus in its generalized form and integral calculus of functions of several variables, developed up to the general formula of Newton–Leibniz–Stokes, thus acquires a certain unity and becomes more self-contained.

More complete information on the textbook and some recommendations for its use in teaching can be found in the translations of the prefaces to the first and second Russian editions.

Moscow, Russia V. Zorich
2003

Preface to the Sixth Russian Edition

On my own behalf and on behalf of future readers, I thank all those, living in different countries, who had the possibility to inform the publisher or me personally about errors (typos, errors, omissions), found in Russian, English, German and Chinese editions of this textbook.

As it turned out, the book has been also very useful to physicists; I am very happy about that. In any case, I really seek to accompany the formal theory with meaningful examples of its application both in mathematics and outside of it.

The sixth edition contains a series of appendices that may be useful to students and lecturers. Firstly, some of the material is actually real lectures (for example, the transcription of two introductory survey lectures for students of first and third semesters), and, secondly, this is some mathematical information (sometimes of current interest, such as the relation between multidimensional geometry and the theory of probability), lying close to the main subject of the textbook.

Moscow, Russia V. Zorich
2011

Prefaces to the Fifth, Fourth, Third and Second Russian Editions

In the fifth edition all misprints of the fourth edition have been corrected.

Moscow, Russia V. Zorich
2006

In the fourth edition all misprints that the author is aware of have been corrected.

Moscow, Russia V. Zorich
2002

The third edition differs from the second only in local corrections (although in one case it also involves the correction of a proof) and in the addition of some problems that seem to me to be useful.

Moscow, Russia V. Zorich
2001

In addition to the correction of all the misprints in the first edition of which the author is aware, the differences between the second edition and the first edition of this book are mainly the following. Certain sections on individual topics – for example, Fourier series and the Fourier transform – have been recast (for the better, I hope). We have included several new examples of applications and new substantive problems relating to various parts of the theory and sometimes significantly extending it. Test questions are given, as well as questions and problems from the midterm examinations. The list of further readings has been expanded.

Further information on the material and some characteristics of this second part of the course are given below in the preface to the first edition.

Moscow, Russia V. Zorich
1998

Preface to the First Russian Edition

The preface to the first part contained a rather detailed characterization of the course as a whole, and hence I confine myself here to some remarks on the content of the second part only.

The basic material of the present volume consists on the one hand of multiple integrals and line and surface integrals, leading to the generalized Stokes' formula and some examples of its application, and on the other hand the machinery of series and integrals depending on a parameter, including Fourier series, the Fourier transform, and the presentation of asymptotic expansions.

Thus, this Part 2 basically conforms to the curriculum of the second year of study in the mathematics departments of universities.

So as not to impose rigid restrictions on the order of presentation of these two major topics during the two semesters, I have discussed them practically independently of each other.

Chapters 9 and 10, with which this book begins, reproduce in compressed and generalized form, essentially all of the most important results that were obtained in the first part concerning continuous and differentiable functions. These chapters are starred and written as an appendix to Part 1. This appendix contains, however, many concepts that play a role in any exposition of analysis to mathematicians. The presence of these two chapters makes the second book formally independent of the first, provided the reader is sufficiently well prepared to get by without the numerous examples and introductory considerations that, in the first part, preceded the formalism discussed here.

The main new material in the book, which is devoted to the integral calculus of several variables, begins in Chap. 11. One who has completed the first part may begin the second part of the course at this point without any loss of continuity in the ideas.

The language of differential forms is explained and used in the discussion of the theory of line and surface integrals. All the basic geometric concepts and analytic constructions that later form a scale of abstract definitions leading to the generalized Stokes' formula are first introduced by using elementary material.

Chapter 15 is devoted to a similar summary exposition of the integration of differential forms on manifolds. I regard this chapter as a very desirable and systematizing supplement to what was expounded and explained using specific objects in the mandatory Chaps. 11–14.

The section on series and integrals depending on a parameter gives, along with the traditional material, some elementary information on asymptotic series and asymptotics of integrals (Chap. 19), since, due to its effectiveness, the latter is an unquestionably useful piece of analytic machinery.

For convenience in orientation, ancillary material or sections that may be omitted on a first reading, are starred.

The numbering of the chapters and figures in this book continues the numbering of the first part.

Biographical information is given here only for those scholars not mentioned in the first part.

As before, for the convenience of the reader, and to shorten the text, the end of a proof is denoted by □. Where convenient, definitions are introduced by the special symbols := or =: (equality by definition), in which the colon stands on the side of the object being defined.

Continuing the tradition of Part 1, a great deal of attention has been paid to both the lucidity and logical clarity of the mathematical constructions themselves and the demonstration of substantive applications in natural science for the theory developed.

Moscow, Russia V. Zorich
1982

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