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Xiwang Dong

Formation and Containment Control for High-order Linear Swarm Systems

Doctoral Thesis accepted by
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 Springer

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- [2] Xiwang Dong, Bocheng Yu, Zongying Shi, Yisheng Zhong, Time-varying formation control for unmanned aerial vehicles: Theories and applications, *IEEE Transactions on Control Systems Technology*, 2015, 23(1): 340–348. (Reproduced with permission)
- [3] Xiwang Dong, Fanlin Meng, Zongying Shi, Geng Lu, Yisheng Zhong, Output containment control for swarm systems with general linear dynamics: A dynamic output feedback approach, *Systems & Control Letters*, 2014, 71(1): 31–37. (Reproduced with permission)
- [4] Xiwang Dong, Jianxiang Xi, Geng Lu, Yisheng Zhong, Containment analysis and design for high-order linear time-invariant singular swarm systems with time delays, *International Journal of Robust and Nonlinear Control*, 2014, 24(7): 1189–1204. (Reproduced with permission)
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- [6] Xiwang Dong, Zongying Shi, Geng Lu, Yisheng Zhong, Time-varying output formation control for high-order linear time-invariant swarm systems, *Information Sciences*, 2015, 298(20): 36–52. (Reproduced with permission)
- [7] Xiwang Dong, Zongying Shi, Geng Lu, Yisheng Zhong, Formation-containment analysis and design for high-order linear time-invariant swarm systems, *International Journal of Robust and Nonlinear Control*, in press, 2014. (Reproduced with permission)
- [8] Xiwang Dong, Jianxiang Xi, Zongying Shi, Yisheng Zhong, Practical consensus for high-order linear time-invariant swarm systems with interaction uncertainties, time-varying delays and external disturbances, *International Journal of Systems Science*, 2013, 44(10): 1843–1856. (Reproduced with permission)

Supervisor's Foreword

In the past decades, researches on swarm systems consisting of multiple subsystems (also called agents) with interactions have attracted much attention in various fields. In engineering, how to design distributed algorithms that rely on only local interactions for a group of agents working together to achieve certain global group behaviors has become an attractive focus in many scientific communities, especially the control and robotics communities. Compared with a single complex agent, multiple simple agents working cooperatively can obtain great benefits which include low cost, high scalability and flexibility, great robustness, and easy maintenance. This gives rise to a very active and exciting research field cooperative control of swarm systems (or multi-agent systems) which has potential applications in many areas such as cooperative control of intelligent transportation systems, distributed control of power systems, cooperation of multiple robots, distributed optimization of networked systems, formation flying of multiple satellites and unmanned aerial vehicles (UAVs), etc.

In this doctoral thesis, Xiwang Dong investigates formation and containment control problems of high-order linear time-invariant swarm systems with consensus control problems, formation control problems, containment control problems, and formation-containment control problems, which are typical cooperative control problems, discussed respectively. Sufficient conditions for swarm systems with time-varying delays, interaction uncertainties, and external disturbances to achieve practical consensus are presented. Necessary and sufficient conditions for swarm systems to achieve time-varying state/output formations and those for state/output formation feasibilities are proposed respectively. Approaches to specify the motion modes of state/output formation references, to expand feasible state/output formation sets, and to design the state/output formation protocols are proposed respectively. Necessary and sufficient conditions for swarm systems to achieve state/output containment and approaches to design the state/output containment protocols are presented. Sufficient conditions for swarm systems to achieve state/output formation-containment and approaches to design the protocols are shown respectively. State/output consensus problems, state/output consensus

tracking problems, state/output formation problems, state/output containment problems, and state formation-containment problems can be regarded as special cases of output formation-containment problems. Theoretical results on formation control are applied to solve the time-varying formation control problems of UAV swarm systems. Autonomous time-varying formation flight experiments using five quadrotor UAVs are performed in outdoor environment.

The above-mentioned results obtained by Xiwang have been appreciated by peer reviewers and editors. For example, the associate editor and reviewers of IEEE Transactions on Control of Network Systems have given the comments that "As all reviewers agree, the paper contains fairly original results" and "The work presented is fairly relevant for control engineering applications as its generality allows to apply it to a considerable pool of scenarios." The reviewers of IEEE Transactions on Control Systems Technology have concluded that the time-varying formation experiments are "remarkable" and "meaningful (recall that most existing work focused on theoretical analysis)." This thesis not only contributes to the theoretical development of cooperative control of swarm systems but also provides practical approaches for the formation and containment control of many swarm systems in engineering.

Beijing
May 2015

Prof. Yisheng Zhong

Abstract

Formation and containment control of swarm systems has broad applications in many fields, such as formation control of unmanned aerial vehicle (UAV) swarm systems, formation control of multiple satellites, and cooperation of multiple robots. This thesis investigates formation and containment control problems of high-order linear time-invariant swarm systems with consensus control problems, formation control problems, containment control problems, and formation-containment control problems discussed respectively. Moreover, theoretical results for formation control of high-order swarm systems are applied to solve the time-varying formation control problems of UAV swarm systems. The main contributions of the thesis are as follow.

- (1) The concept of practical consensus is proposed. Sufficient conditions for swarm systems with non-uniform time-varying delays, interaction uncertainties, and time-varying external disturbances which can belong to L_2 or L_∞ to achieve state practical consensus are presented. Explicit expressions of the state practical consensus function and the consensus error bound are also given.
- (2) Time-varying state formation control problems for high-order swarm systems with time delays and time-varying output formation control problems for high-order swarm systems are studied respectively. Necessary and sufficient conditions for swarm systems to achieve time-varying state/output formations, necessary and sufficient conditions for state/output formation feasibilities, and explicit expressions of state/output formation reference functions are presented. Approaches to specify the motion modes of state/output formation references, approaches to expand feasible state/output formation sets, and approaches to design the state/output formation protocols are proposed respectively. Necessary and sufficient conditions for UAV swarm systems to achieve time-varying formations and approaches to design the protocol are proposed. Autonomous time-varying formation experiments using five quadrotor UAVs are performed in outdoor environment to demonstrate the theoretical results.

- (3) An output containment protocol is proposed based on the dynamic output feedback for high-order swarm systems. Necessary and sufficient conditions for swarm systems to achieve output containment and approaches to design the output containment protocol are presented. For high-order singular swarm systems with time delays, sufficient conditions to achieve state containment are proposed and an approach with less computational complexity to design the protocol is given. It is shown that containment results for singular swarm systems can be applied to solve the consensus tracking problems for singular swarm systems, state containment, and consensus tracking problems for normal swarm systems.
- (4) For formation-containment problems of high-order swarm systems, sufficient conditions to achieve state/output formation-containment and approaches to design the protocols are proposed respectively. Necessary and sufficient conditions for high-order swarm systems to achieve state containment are presented as special cases. It is pointed out that state/output consensus problems, state/output consensus tracking problems, state/output formation problems, state/output containment problems, and state formation-containment problems can be regarded as special cases of output formation-containment problems.

Keywords Swarm system • Consensus control • Formation control • Containment control • Formation-containment control

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Acronyms

AUV	Autonomous underwater vehicle
BIBS	Bounded input and bounded state
CCL	Core complementarity linearization
CEP	Circular error probable
DoF	Degree-of-freedom
FCS	Flight control system
GCS	Ground control station
GPS	Global positioning system
ILMI	Interactive linear matrix inequality
LMI	Linear matrix inequality
LTI	Linear time-invariant
PBH	Popov-Belevitch-Hautus
PD	Proportional-derivative
QMI	Quadratic matrix inequality
UAV	Unmanned aerial vehicle

Notations

G	Graph
L	Laplacian matrix
\mathbb{R}^n	Set of $n \times 1$ real vectors
\mathbb{R}^{Nn}	Set of $Nn \times 1$ real vectors
$\mathbb{R}^{N \times n}$	Set of $N \times n$ real matrices
$\mathbb{C}^{N \times n}$	Set of $N \times n$ complex matrices
I	Identity matrix with appropriate dimension
$\mathbf{1}$	Column vector of all ones
e_i	Column vector with 1 as its i th component and 0 elsewhere
0	0 number, 0 vector or 0 matrix with appropriate dimension
\otimes	Kronecker product
\oplus	Direct sum
\equiv	Identically equal
\Rightarrow	Implies
$\ x\ $	2-norm of a real vector x
$\text{rank}(A)$	Rank of matrix A
Λ_A	Block diagonal matrix with diagonal blocks A
$\text{Re}(\lambda)$	Real part of number λ
$\text{Im}(\lambda)$	Imaginary part of number λ
Ψ_λ	Matrix $\Psi_\lambda = \begin{bmatrix} \text{Re}(\lambda)I & -\text{Im}(\lambda)I \\ \text{Im}(\lambda)I & \text{Re}(\lambda)I \end{bmatrix}$
A^H	Hermitian transpose of matrix A
A^T	Transpose of matrix A
\mathcal{L}	Subscript set of leaders
\mathcal{F}	Subscript set of followers
$*$	Symmetry term in symmetric block matrix
Θ	A pseudorandom value with a uniform distribution on the interval $(0, 1)$