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Front Tracking for Hyperbolic Conservation Laws

2nd Edition

Springer

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In memory of Raphael, who started it all

Preface to the Second Edition

On general grounds I deprecate prefaces.[1]
— Winston Churchill

In this edition we have added the following new material: In Chapt. 1 we have added a section on linear equations, which allows us to present some of the material in the book in the simpler linear setting. In Chapt. 2 we have made some changes in the presentation of Kružkov's fundamental doubling of variables method. In Chapt. 3 on finite difference methods the focus has been changed to finite volume methods. A section on higher-order schemes has been added. The section on measure-valued solutions has been rewritten. The main existence theorem in Chapt. 4, Theorem 4.3, now resembles the one-dimensional case. The presentation of the solution of the Riemann problem for systems in Chapt. 5 has been supplemented by the complete solution of the Riemann problem for the 3×3 Euler equations of gas dynamics. The solution of the Cauchy problem for systems in Chapt. 6 has been rewritten and simplified. We have added a new chapter, Chapt. 8, on one-dimensional scalar conservation laws where the flux function depends explicitly on space in a discontinuous manner.

In addition, we have corrected mistakes that we have discovered. Furthermore, we have polished the presentation in several places, and new exercises have been added. We are grateful to those who have given us feedback, in particular G.M. Coclite, U. Skre Fjordholm, F. Gossler, K. Grunert, H. Hanche-Olsen, Espen R. Jakobsen, Qifan Li, S. May, A. Nordli, X. Raynaud, M. Rejske, O. Sete, K. Varholm, and F. Weber. The extensive help from Olivier Buffet in setting up the flip cartoons is much appreciated. We are very grateful to David Kramer for careful copyediting.

[1] in *The Story of the Malakand Field Force: An Episode of Frontier War* (*1898*).

Preface to the First Edition

Все счастливые семьи похожи друг на друга,
каждая несчастливая семья несчастлива по-своему.[2]
— Лев Толстой, Анна Каренина (1875)

While it is not strictly speaking true that all linear partial differential equations are the same, the theory that encompasses these equations can be considered well developed (and these are the happy families). Large classes of linear partial differential equations can be studied using linear functional analysis, which was developed in part as a tool to investigate important linear differential equations.

In contrast to the well-understood (and well-studied) classes of linear partial differential equations, each nonlinear equation presents its own particular difficulties. Nevertheless, over the last forty years some rather general classes of nonlinear partial differential equations have been studied and at least partly understood. These include the theory of viscosity solutions for Hamilton–Jacobi equations, the theory of Korteweg–de Vries equations, as well as the theory of hyperbolic conservation laws.

The purpose of this book is to present the modern theory of hyperbolic conservation laws in a largely self-contained manner. In contrast to the modern theory of linear partial differential equations, the mathematician interested in nonlinear hyperbolic conservation laws does not have to cover a large body of general theory to understand the results. Therefore, to follow the presentation in this book (with some minor exceptions), the reader does not have to be familiar with many complicated function spaces, nor does he or she have to know much theory of linear partial differential equations.

The methods used in this book are almost exclusively constructive, and largely based on the front-tracking construction. We feel that this gives the reader an intuitive feeling for the nonlinear phenomena that are described by conservation laws. In addition, front tracking is a viable numerical tool, and our book is also suitable for practical scientists interested in computations.

We focus on scalar conservation laws in several space dimensions and systems of hyperbolic conservation laws in one space dimension. In the scalar case we first discuss the one-dimensional case before we consider its multidimensional generalization. Multidimensional systems will not be treated. For multidimensional

[2] All happy families resemble one another, but every unhappy family is unhappy in its own way (Leo Tolstoy, *Anna Karenina*).

equations we combine front tracking with the method of dimensional splitting. We have included a chapter on standard difference methods that provides a brief introduction to the fundamentals of difference methods for conservation laws.

This book has grown out of courses we have given over some years: full-semester courses at the Norwegian University of Science and Technology, the University of Oslo, and Eidgenössische Technische Hochschule Zürich (ETH), as well as shorter courses at Universität Kaiserslautern, S.I.S.S.A., Trieste, and Helsinki University of Technology.

We have taught this material for graduate and advanced undergraduate students. A solid background in real analysis and integration theory is an advantage, but key results concerning compactness and functions of bounded variation are proved in Appendix A.

Our main audience consists of students and researchers interested in analytical properties as well as numerical techniques for hyperbolic conservation laws.

We have benefited from the kind advice and careful proofreading of various versions of this manuscript by several friends and colleagues, among them Petter I. Gustafson, Runar Holdahl, Helge Kristian Jenssen, Kenneth H. Karlsen, Odd Kolbjørnsen, Kjetil Magnus Larsen, Knut-Andreas Lie, Achim Schroll. Special thanks are due to Harald Hanche-Olsen, who has helped us on several occasions with both mathematical and TEX-nical issues.

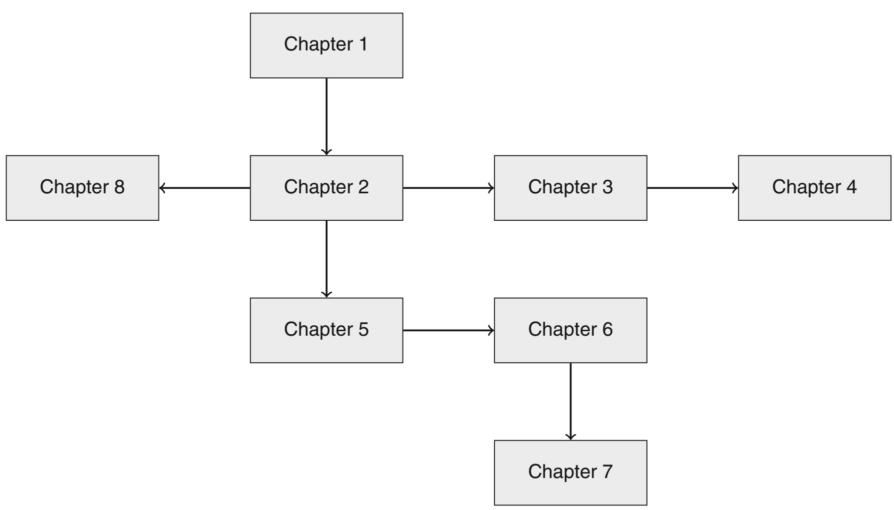
Our research has been supported in part by the BeMatA program of the Research Council of Norway.

A list of corrections can be found at

http://www.math.ntnu.no/~holden/FrontBook/

Whenever you find an error, please send us an email about it.

The logical interdependence of the material in this book is depicted in the diagram below. The main line, Chapts. 1, 2, 5–7, has most of the emphasis on the theory for systems of conservation laws in one space dimension. Another possible track is Chapts. 1–4, with emphasis on numerical methods and theory for scalar equations in one and several space dimensions. Chapt. 8, on the theory for one-dimensional scalar conservation laws with spatially depending flux function, requires only Chapts. 1 and 2.



Dependencies among the chapters

Flip Cartoons[3]

Well, the silent pictures were the purest form of cinema.
— Alfred Hitchcock

We have included four flip cartoons in the book: At the bottom of the odd-numbered pages (starting from the back) you see the solution of the equation

$$u_t + \frac{1}{3}(u^3)_x = 0, \quad u|_{t=0} = \cos(\pi x),$$

using a second-order finite difference method, more specifically, the Lax–Wendroff method with minmod limiter; see (3.43). On the bottom of the even-numbered pages (starting from the front) you see the fronts in the (x, t)-plane for the same problem; see (2.44).

At at top of the odd-numbered pages (starting from the back) you see the solution of the Euler equations (5.150) with $\gamma = 1.4$. The initial data are

$$p(x, 0) = \begin{cases} 3 & \text{for } |x| \le 0.5, \\ 1 & \text{otherwise,} \end{cases} \qquad \rho(x, 0) = \begin{cases} 2.5 & \text{for } |x| \le 0.25, \\ 1 & \text{otherwise,} \end{cases} \qquad v(x, 0) = 0,$$

and the data are extended periodically outside the interval $(-1, 1)$. The pressure p is displayed for $t \in [0, 1]$, and the solution is obtained using the Godunov method with a Roe approximate Riemann solver. We use $\Delta x = 1/250$. On the bottom of the even-numbered pages (starting from the front) you see the fronts in the (x, t)-plane for the same problem; see (6.9).

We do not want now and we shall never want the human voice with our films.
— D.W. Griffiths (1875–1948), movie pioneer

As for readers of the eBook, we refer to Springer's web site where one can watch the flip cartoons.

Maybe eBooks are going to take over, one day, but not until those whizzkids in Silicon Valley invent a way to bend the corners, fold the spine, yellow the pages, add a coffee ring or two and allow the plastic tablet to fall open at a favorite page.
— R.T. Davies, in foreword to D. Adams's The Hitchhiker's Guide to the Galaxy

[3] Assistance from Olivier Buffet is much appreciated.

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