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Rudi Zagst

Interest-Rate Management

Springer

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Meinen Eltern !

Als ich klein und hilflos war,
gabt Ihr mir Geborgenheit und Schutz.
Als ich größer wurde und Weisheit suchte,
halft Ihr mir, sie zu finden.
Als ich meinen Weg suchen mußte,
habt Ihr mich gehen lassen.
Als ich ihn fand,
halft Ihr mir, meine Ziele zu erreichen.
Als ich dieses Buch schrieb,
habe ich gemerkt, daß ich mich viel zu selten bei Euch bedankt habe.

Vielen Dank dafür,
daß ich mich stets auf Eure Liebe und Treue verlassen konnte.
Es war und ist ein Glück, daß es Euch gibt !

Preface

Who gains all his ends did set the level too low.

Although the history of trading on financial markets started a long and possibly not exactly definable time ago, most financial analysts agree that the core of mathematical finance dates back to the year 1973. Not only did the world's first option exchange open its doors in Chicago in that year but Black and Scholes published their pioneering paper [BS73] on the pricing and hedging of contingent claims. Since then their explicit pricing formula has become the market standard for pricing European stock options and related financial derivatives. In contrast to the equity market, no comparable model is accepted as standard for the interest-rate market as a whole. One of the reasons is that interest-rate derivatives usually depend on the change of a complete yield curve rather than only one single interest rate. This complicates the pricing of these products as well as the process of managing their market risk in an essential way. Consequently, a large number of interest-rate models have appeared in the literature using one or more factors to explain the potential changes of the yield curve. Beside the Black ([Bla76]) and the Heath-Jarrow-Morton model ([HJM92]) which are widely used in practice, the LIBOR and swap market models introduced by Brace, Gątarek, and Musiela [BGM97], Miltersen, Sandmann, and Sondermann [MSS97], and Jamshidian [Jam98] are among the most promising ones. However, up to now, none of the existing models can be considered as more than a standard for a sub-market such as the cap or swap market.

Inconsistencies usually appear once these models are to be used for pricing other interest-rate derivatives jointly.

To understand all the different interest-rate models, and to be able to develop new models, one needs a thorough background in stochastic calculus and financial mathematics. Excellent books for the advanced reader in this field are, e.g., Lamberton and Lapeyre [LL97], Musiela and Rutkowski [MR97], or Øksendal [Øks98]. On the other hand, there are also books written for a more economics oriented readership. Very good representatives, e.g., are Hull [Hul00] or Baxter and Rennie [BR96]. Books aiming for a middle way between these two species are, for instance, the excellent texts of Bingham and Kiesel [BK98] or Korn and Korn [KK99]. However, none of these books addresses the complete financial engineering process, i.e. modelling, pricing, hedging as well as medium and long-term risk and asset management. And indeed, this is the main reason

...why I have written this book.

In many discussions with my students at the universities of Ulm, Augsburg and Munich, as well as during my courses and consulting activities for banks, insurance companies, and other financial institutions, the question appeared of whether there is a book describing the whole process - from mathematical modelling and pricing to the risk and asset management of a complete portfolio or trading book. A list of different books has been the best advice I could give. Then some years ago, when we were discussing this very topic during a car ride from Munich to Ulm, a good friend of mine, Dr. Gerhard Scheuenstuhl, encouraged me to close this gap by writing a book about both sides of the coin, the mathematical modelling and the risk management. Of course, covering the whole story would have been a daunting task and would have resulted in many more pages than you hold in your hand. The background material of stochastic calculus had to be restricted, as well as the number of models and derivatives being discussed and the topics covering risk management issues. However, it was the aim of the author to give an insight into the long road of modelling an interest-rate market, mark-to-market a selection of interest-rate derivatives and simulate their future value using the market model (mark-to-future), as well as deriving valuable risk numbers applied within a reliable risk management process. So after all this,

...what is this book about?

We begin with an overview of the most important mathematical tools for describing financial markets, i.e. stochastic processes and martingales. These methods are applied to modelling a financial market and, in particular, to modelling an interest-rate market. We will learn about different interest-rate models driving the prices of financial assets, as well as different methods for pricing interest-rate derivatives. These are a pure application of the martingale theory, an application of the theory of Green's

functions, and an application of the important change-of-numéraire technique. Each of these methods is applied within a specific model showing the wide spectrum of possibilities we have in the evaluation of financial products. However, it was not possible to cover all or even most of the existing interest-rate models available in the literature. Also it was not the intention of the author to compete with the books of Lamberton and Lapeyre, Musiela and Rutkowski, or Øksendal which go much further into the mathematical details than this book. Rather, we follow the books of Bingham and Kiesel or Korn and Korn on their middle way through mathematical finance before we leave their path to aim for the measuring and management of market risk. Short- and long-term risk measures will be discussed, as well as a selection of optimization problems, which are solved to maximize the performance of a portfolio under limited downside risk. Now you may ask

... for whom have I written this book?

This book is written for students, researchers, and practitioners who want to get an insight into the modelling of interest-rate markets as well as the pricing and management of interest-rate derivatives. Chapters 2 and 3 of the book give a rigorous overview of the mathematics of financial markets. They present the most important tools needed to describe the movement of market prices and define the theoretical framework for the pricing and hedging of contingent claims. A basic knowledge of probability theory and a certain quantitative background are recommended as prerequisites. Those looking for a crash course in stochastic processes and the modelling of financial markets will hopefully find this part to be a valuable source. However, if you are already familiar with the Itô calculus and the methods for pricing and hedging financial derivatives, you could immediately start with Chapter 4. Here and in Section 5 we focus on the modelling and pricing in interest-rate markets. If you already know most of the specific interest-rate models you may just glance over Section 4, which shows how an interest-rate market can be embedded in the financial market framework of Section 3. In Chapter 5 we describe the most popular interest-rate derivatives, and show how they can be priced using a specific interest-rate model. Because the trading and risk management of derivatives are dominated mainly by the application of specific models and techniques, the style of the book will also change gradually to a more economics oriented one. Real-world applications have to take care of market conventions such as daycounts or special rates which are, from a mathematical point of view, not too much of a deal but which may vary between different markets and result in misleading risk numbers and prices once they are ignored. Chapters 6 and 7 are intended to give an insight into the practical application of interest-rate models to the risk and portfolio management of interest-rate derivatives. They cover a selection of short- and long-term-oriented risk measures as well as comprehensive case studies based on real market data. We hope

those interested in mark-to-future simulations, specific risk numbers, and their use for risk management will enjoy reading these chapters. Should you be more practice-oriented, you will not need a full understanding of stochastic calculus and martingale theory for Sections 5 to 7. A basic understanding of the main results of Chapters 2 to 4 will be sufficient. However, those getting fascinated with the potential of financial modelling may look for the "math necessities" within these chapters. Since all parts of the book have been used in teaching mathematical finance, financial engineering and risk management at different universities, this textbook may also serve as a valuable source for graduate and PhD students in mathematics or finance who want to acquire some knowledge of financial markets and risk management.

A final word!

Satisfying the needs of both practitioners and researchers is always a hard and sometimes too hard a problem to solve. A gap still exists between these two worlds, and accordingly, there remains a gap between the corresponding parts of the book. I have tried to make this gap as small as possible. I also had to restrict myself to a brief overview of stochastic calculus, where a lot more could have been said and proved. On account of the idea and limited size of the book, I had to select a small variety of interest-rate models and discuss their pricing effects rather than show for which market which model works best. The reader interested in this question may, for instance, refer to Brigo and Mercurio [BM01]. I also could not cover all risk-management topics, since this is a boundless field in its own right. Therefore I surely didn't gain all my intended ends. Nevertheless, I hope that I have succeeded in finding a good middle way to describe the process from mathematical modelling and pricing to the risk and asset management of interest-rate derivatives portfolios. Whether I have reached this target will be for the reader to judge.

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Munich, January 2002 Rudi Zagst

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