

Part III

**Differential Geometry**

In this purely mathematical part, we develop the most important concepts and results of differential geometry which are needed for general relativity theory.

The presentation differs little from that in many contemporary mathematical text books. The language of modern differential geometry and the “intrinsic” calculus on manifolds are now frequently used by workers in the field of general relativity and are beginning to appear in textbooks on the subject. This has a number of advantages, such as:

- a) It enables one to read the mathematical literature and make use of the results to attack physical problems.
- b) The fundamental concepts, such as differentiable manifolds, tensor fields, affine connection, and so on, adopt a clear and intrinsic formulation.
- c) Physical statements and conceptual problems are not confused by the dependence on the choice of coordinates. At the same time, the role of distinguished coordinates in the physical applications is clarified. For example, these can be adapted to symmetry properties of the system.
- d) The exterior calculus of differential forms is a very powerful method for practical calculations; one often finds the results faster than with older methods.

Space does not allow us to always give complete proofs and sufficient motivation. In those cases, we give detailed references to the literature (see [30]-[48]) where these can be found. In the chapter entitled “Some Details” the reader can find some of the proofs not given in the main body of the text. Many readers will have the requisite mathematical knowledge to skip this part after familiarizing themselves with our notation (which is quite standard). This is best done by looking at the collection of important formulas in Appendix D. Readers who have no previous knowledge in differential geometry should first study all of Chaps. 9 and 10 and then go directly to Chap. 13, and absorb Sects. 13.1-13.6. These contain the most important tools for GR and form a selfcontained part. Other sections may be studied when they are needed.